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INTRODUCTION

THE object of this work is to cover a field not before set forth in any book to the knowledge of the writers, namely, the calculation of frog-work and special curves for use in the shop. Many problems on curves have been omitted that are not pertinent to this work, but, on the other hand, many new ones are set forth, and some of the old ones with the methods changed.

In the problems on "clearance curves" approximate methods are used, as exact figuring would be long and laborious without the results being any more useful.

The subject of transition curves is not a new one, nor is the particular spiral treated in this book new, but the treatment and tables are quite different from others and the spiral is made flexible, something hitherto lacking.

The tables on the reduction of inches to decimals of a foot, middle ordinates of ten-foot chords, railroad curves and chords, may be found in many "Pocket Books," but are here introduced for the saving of time to the user. "Logarithms in Common Use" will save turning many pages, and the "Table for Laying Curves" was designed for shop assembly use. Other tables are explained in the text.

In making calculations for special work it is well to follow a general system, not only for one's convenience while making the calculation, but also for future reference. Such a system is the placing on the paper of the different parts of the whole calculation, that one may immediately find the part required. Do not be tied to logarithms, use natural functions, slide rule, tables for squares, etc., as the occasion may find easiest.

SLIDE RULE

It is surprising what a small number of calculators habitually use the Mannheim slide rule. In making computations where the result sought is in four or less places of digits, for approximate values and for checking, the slide rule is one of the handiest inventions and an incalculable amount of time may be saved by making use of it. For the benefit of those who are not familiar with it, a few words will explain the principle and method of operation.

Place two similarly divided rules side by side so that they read from zero in the same direction; to add three and five, move the zero of the lower rule to the three of the upper, and opposite the five of the lower will be found eight on the upper rule. Now in multiplying two numbers by the use of logs, we add the logs of the numbers and look up the number opposite the sum of the logs for a product. Say we wish to multiply three by five:

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$$\begin{aligned}\text{Log } 3 &= .477\\ \text{Log } 5 &= \underline{.699}\\ \text{Log } 15 &= 1.176\end{aligned}$$

We could have added the .477 and .699 on the rules as before and obtained 1.176; but, if 3, 5, and 15 had been marked against the distances .477, .699, and 1.176 respectively on the rules, the product would have been obtained immediately. This is the principle of the slide rule: the figures on the rule correspond with the logarithmic distances and thus the adding the logarithmic distances multiplies the indicated figures.

On the slide rule the numbers on the lower side run from 1 to 10, and on the upper side from 1 to 100, or from 1 to 10, and repeat so that the space from 1 to 10 on the upper side is half the space from 1 to 10 on the lower side. Since, when the log of a number is multiplied by two the log of the square of the number is obtained, the numbers on the upper side of the rule are squares of the numbers opposite on the lower side. In taking the square root of a number on the rule note if there are an even or an odd number of digits to the left of the decimal point, for the square root of 36, for instance, contains different digits than the square root of 360. For an odd number of digits use the left-hand half of rule, or from 1 to 10, and for an even number of digits use the right half, or from 10 to 100. This same principle is true in fixing the position of the decimal in multiplying and dividing, for when two numbers are multiplied the product contains as many, or one less than as many, digits as the two numbers together contain; for example, to multiply 3760 by 81,407 set the end of the slide on 37.6, the indicator on 8.141 of the slide, and read 3.06 on the rule. Now the sum of the digits of the two numbers is 9, and since the result comes odd the result must be 306,000,000. In using the lower side of the rule, when the result has the same number of digits as the sum of the digits in the two numbers the slide projects to the left, if one less digit it projects to the right. Either end of the slide may be used in setting and at any time the ends of the slide may be changed by moving the indicator to one end of the slide and then moving the other end to the indicator.

In selecting the slide rule be sure that one side of the slide gives trigonometric functions (sines and tangents). If the slide is reversed the process is the same as for numbers; if the slide is not reversed, to multiply by the sine set the sine on the mark on the under side of the rule and on the upper half of rule opposite the number read the result on the slide; to divide by the sine, set the sine as before and opposite the number on the slide read the result on the rule; to divide a number by a number and obtain the sine, set the dividend on the slide opposite the divisor on the rule and read the sine at the mark. Tangents cannot be worked this way. Tangents smaller than 5° 43′ are not given, but small tangents and sines vary so little that sines of small angles may be used for tangents. To multiply a num[illegible] by a tangent of an angle greater than 45° divide the number by the tangent of the complement of the angle. For the cosine use the sine of the complement of the angle.

To divide one number by another and obtain the sine or tangent of an angle when the slide is reversed, set the indicator on the dividend, the end of the slide on the divisor, and read the angle on the slide at the indicator.

To multiply a number by the sine of a large angle. As the divisions of the sines are hard to read closely near 90°, it is better to make a correction to the number by making use of the following formula:

$$\text{Sin } A = \cos (90^\circ - A) = 1 - \text{vers } (90^\circ - A) = 1 - 2 \sin^2 \tfrac{1}{2} (90^\circ - A) \quad . \; . \; . \; . \; . \; . \; (1)$$

It will be found convenient to make a mark on the slide in the sine divisions opposite the two on the rule for use in this calculation.

To divide a number by the sine of a large angle:

$$\frac{1}{\sin A} = \text{cosec } A = \text{sec. } (90^\circ - A) = 1 + \text{exsec } (90^\circ - A) = 1 + \tan (90^\circ - A) \tan \tfrac{1}{2} (90^\circ - A) \quad . \; . \; . \; . \; (2)$$

In a right-angle triangle let b = base, a = altitude, and h = hypothenuse. $h^2 = b^2 + a^2$.

$$h = b + \frac{a^2}{2b} + \; . \; . \; . \text{ and } b = \text{h} - \frac{a^2}{2h} - \; . \; . \; . \; . \; . \; . \; (3)$$

If a is small in comparison with b and h, these two formulæ give, results very close.

To obtain circular arcs, make a mark on the slide at 57.30. Reducing any minutes to decimals of a degree, set the mark at the radial length on the rule and opposite the number of degrees taken on the slide read the length of arc on the rule.

If R is the radius, c = chord and a = arc in a flat arc. Let y = middle ordinate.

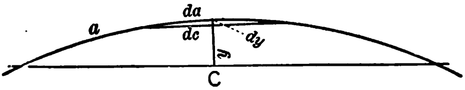

FIG. 1

$$y = \frac{c^2}{8\,R} \text{ (approx.)}$$

$$dy = \frac{cdc}{4\,R}$$

Assume an indefinitely small arc, da, with chord, dc, and middle ordinate, dy. These quantities correspond respectively with $2l$, [illegible] and a of (3).

$$da = dc + \frac{2(dy)^2}{dc} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(from 3)}$$

Substituting the value of *dy* first found,

$$da = dc + \frac{2c^2(dc)^2}{16\,\mathrm{R}^2\mathrm{dc}} = dc + \frac{c^2dc}{8\,\mathrm{R}^2}$$

Integrating,

$$a = c + \frac{C^3}{24\ R^2}\,\text{(approx.)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Calling the half chord or sine, *S*, and calling *a* the arc opposite *S*, we may write from (4) $a = S + \frac{S^3}{6\,R^2}$ but this would apply only to flat arcs. For any arc the equation becomes

$$a = S + \frac{S^3}{K\,R^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which *K* is a quantity varying with the angle of the arc.

Angle	Sin $= \frac{S}{R}$	K	Angle	Sin $= \frac{S}{R}$	K	Angle	Sin $= \frac{S}{R}$	K
3°	.052	6.00	15°	.259	5.82	23	.391	5.58
5	.087	5.98	16	.276	5.79	24	.407	5.54
7	.122	5.96	17	.292	5.77	25	.423	5.50
9	.156	5.93	18	.309	5.74	26	.438	5.46
11	.191	5.90	19	.326	5.71	27	.454	5.42
12	.208	5.88	20	.342	5.68	28	.469	5.38
13	.225	5.86	21	.358	5.65	29	.485	5.34
14	.242	5.84	22	.375	5.61	30	.500	5.30

To use this formula on the slide rule, on the lower side divide *S* by *R*, which, when transferred to the upper side, is $\frac{S^2}{R^2}$, move the indicator to *S* on the slide on the upper half and we have $\frac{S^3}{R^2}$; at the end of the slide on lower side read $\frac{S}{R} = \sin A$, in the table opposite $\frac{S}{R}$ read *K*, set *K* on the slide at the indicator and at the end of the slide read $\frac{S^3}{KR^2}$ which, added to *S*, gives *a*.

It will be found convenient to make marks on the lower side of the rule under the regular divisions at 1.39, 1.91, 2.33, 2.70, 3.31, [illegible]80, 4.23, 4.62, and 5.00, marking numbers at these marks respect- [illegible] 5.95, 5.90, 5.85, 5.80, 5.70, 5.60, 5.50, 5.40, and 5.30. This [illegible]he necessity of referring to the table.

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When three sides are given, use the formulæ for the half tangents.

$$\text{Tan } \tfrac{1}{2} A = \sqrt{\frac{(S-b)(S-c)}{s(S-a)}} = \frac{1}{s-a}\sqrt{\frac{(S-a)(S-b)(S-c)}{s}}$$

$$\text{Tan } \tfrac{1}{2} B = \frac{1}{S-b}\sqrt{\frac{(S-a)(S-b)(S-c)}{S}}$$

$$\text{Tan } \tfrac{1}{2} C = \frac{1}{S-c}\sqrt{\frac{(S-a)(S-b)(S-c)}{S}}$$

The quantity under the radical is a constant for the formulæ.

Given $a = 175.412$, $b = 102.355$, $c = 77.355$.

$$\begin{array}{lrcl}
 & 175.412 & & \\
 & 102.355 & & \\
 & \underline{77.355} & & \\
 & 2)\underline{355.122} & & \\
\text{colog} & 177.561 & = & 7.7506525 \\
\text{log} & 2.149 & = & 0.3322364 \\
\text{log} & 75.206 & = & 1.8762525 \\
\text{log} & 100.206 & = & \underline{2.0008938} \\
 & & 2) & \underline{1.9600352} \\
 & & & 0.9800176 \\
\text{colog} & 2.149 & = & 9.6677636 \\
\text{''} & 75.206 & = & 8.1237475 \\
\text{''} & 100.206 & = & \underline{7.9991062} \\
 & 77^\circ\ 19'\ 07'' & & 0.6477812 \\
 & 7\ \ 14\ \ 14 & & 9.1037651 \\
 & \underline{5\ \ 26\ \ 39} & & 8.9791238 \\
A & 154^\circ\ 38'\ 14'' & & \\
B & 14\ \ \ 28\ \ 28 & & \\
C & 10\ \ \ 53\ \ 18 & &
\end{array}$$

The formula, $a^2 = b^2 + c^2 - 2\,bc \cos A$, is of no great value in actual computation, but is useful in working out problems, as will be seen later. It is well to remember that cos. $(180^\circ - A) = -$ cos. A, and where this happens the formula becomes $a^2 = b^2 + c^2 + 2bc$ cos. A.

To figure circular arcs, add the log. of the number of seconds in the angle to $4.6855749 - 10$, the log. arc of one second. For example:

$$\begin{array}{rcl}
15^\circ\ 42'\ 37'' & = & 56557'' \\
\log 56557 & = & 4.7524864 \\
\text{'' arc } 1'' & = & \underline{4.6855749} \\
\log \text{arc } 15^\circ\ 42'\ 37'' & = & 9.4380613
\end{array}$$

[illegible] bottoms of the pages in Bruhns' and in Vega's tables are found the reduction of degrees and minutes to seconds, but it is more convenient to mark in red ink the degrees and minutes opposite its value in seconds for each minute in the margin of the book. There will be a different minute every sixth line; thus every line is opposite an even ten seconds and the odd seconds are found in the columns. For example, opposite 3000 mark 8° 20′, at 3006 mark 8° 21′, at 3012 mark 8° 22′, etc. The time thus expended will be found to be well spent. It will be found that the main body of the table runs from 2° 46′ 40″ to 27° 46′ 40″. When the arc of an angle outside these limits is required, divide or multiply the angle by 10 and add or subtract one to the characteristic of log arc 1″; as,

$$
\begin{array}{llcr}
 & 10\overline{)46^\circ\ 23'\ 30''} & & \\
\text{log arc} & 4\ 38\ 21 & = & 4.2227425 \\
 & & & \underline{5.6855749} \\
\text{log arc } 46^\circ\ 23'\ 30'' & & = & 9.9083174
\end{array}
$$

It is well to keep the plot along with the calculations, as it serves to show what further computations are necessary. The figures must at once be put on the plot, and scaling is a good check on the calculations. The slide rule should be constantly used in checking.

Plot all angles from the table of chords.

CURVES

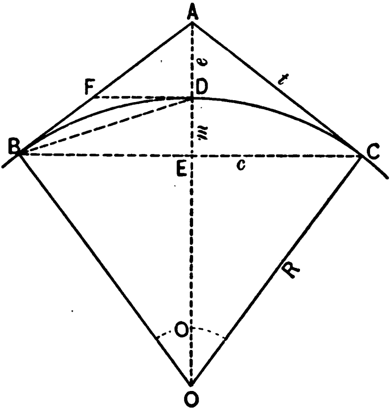

FIG. 2

1. (Fig. 2) $t = R \tan \frac{1}{2} O$ . . . . (6)
2. $R = t \cot \frac{1}{2} O$ . . . . (7)
3. $R = \frac{c}{2} \operatorname{cosec} \frac{1}{2} O$ . . . . (8)
4. $R = \frac{e}{ex} \sec \frac{1}{2} O$ . . . .

10. (Fig. 5) To pass a circular curve through two points, D and C, and tangent to a given line, TB.

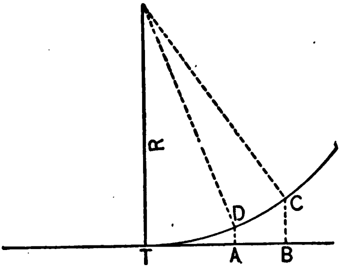

FIG. 5

Figure the distance $AB = a$, and the offsets $AD = b$ and $BC = c$. Let $TA = x$.

From 17, $R = \frac{x^2}{2b} + \frac{b}{2} = \frac{(x+a)^2}{2c} + \frac{c}{2}$

$(c - b)\, x^2 - 2\, abx = bc^2 + a^2\, b - b^2\, c$

$$x = \frac{ab}{c-b} + \sqrt{bc + a\left(\frac{ab}{c-b}\right) + \left(\frac{ab}{c-b}\right)^2} \quad . \quad . \quad (18)$$

$$R = \frac{x^2}{2b} + \frac{b}{2} \quad (14)$$

11. (Fig. 6) To pass a circular curve tangent to a straight line, AD, at a given point, A, and tangent to a given curve. Let C be the center of the given curve and $CB = r$, its radius.

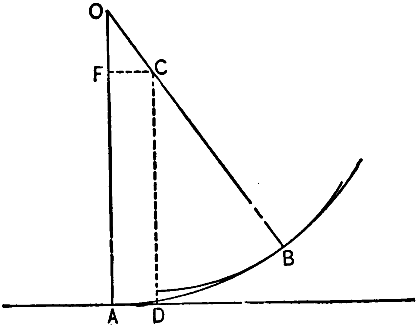

FIG. 6

CASE 1. Where the given line is outside the given curve and the curves do not reverse,

Figure the co-ordinates $CD = a$ and $AD = b$.

Let R = the radius of the curve sought.

$$OC = R - r$$
$$FC = AD = b$$
$$OF = R - a$$
$$(R - r)^2 = (R - a)^2 + b^2$$
$$R = \frac{a + r}{2} + \frac{b^2}{2(a - r)}$$

CASE 2. (Fig. 7) Where the line is outside the given curve and the curves reverse.

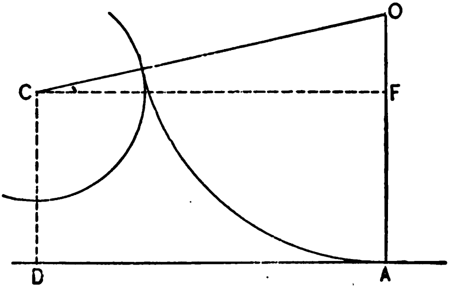

FIG. 7

Figure co-ordinates $CD = a$, and $AD = b$.

Let R = the radius of the curve sought.

$$OC = R + r$$
$$FC = b$$
$$OF = a - R$$
$$(R + r)^2 = (a - R)^2 + b^2$$
$$R = \frac{a - r}{2} + \frac{b^2}{2(r + a)}$$

CASE 3. (Fig. 8) Where the given line cuts the given curve.

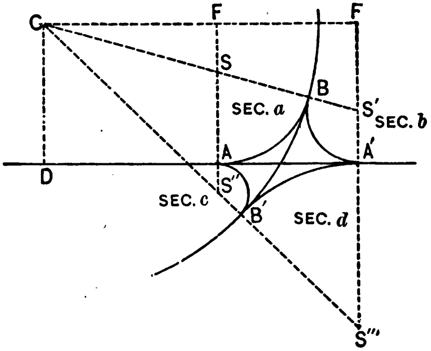

FIG. 8

Figure the co-ordinates $CD = a$ and $DA = b$.
It will be seen in the figure that there are four sections.
In section a, $FS = a - R$, $CF = b$, and $CS = r - R$.
By the same reasoning as before,

$$R = \frac{r + a}{2} - \frac{b^2}{2(r - a)}$$

In section b, $F'S' = a - R$, $CF' = b$, and $CS' = r + R$.

$$R = \frac{b^2}{2(r + a)} - \frac{r - a}{2}$$

In section c, $FS'' = a + R$, $CF = b$, and $CS'' = r - R$.

$$R = \frac{r - a}{2} - \frac{b^2}{2(r + a)}$$

In section d, $F'S''' = a + R$, $CF' = b$, and $CS''' = r + R$.

$$R = \frac{b^2}{2(r - a)} - \frac{r + a}{2}$$

12. (Fig. 9) To pass a circular curve tangent to a straight line, AD, and tangent to a given curve at a given point B.

Let C be the center of the given curve and $CB = r$, its radius.

Let R = the radius of the curve sought.

CASE 1. — Where the given line is outside the given curve and the curves do not reverse.

Figure $CD = a$,
$DE = b$,
and $EB = c$

$$\text{vers } BCD = \frac{r - a + c}{r}$$

$$AOB = BCD$$

$$R = \frac{c}{\text{vers } AOB}$$

$$R = \frac{cr}{r - a + c}$$

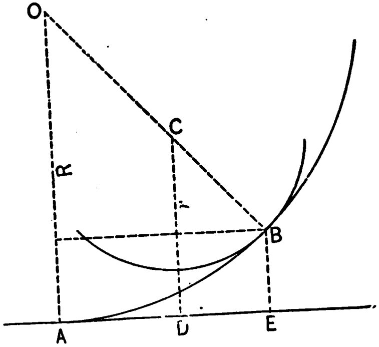

FIG. 9

CASE 2. (Fig. 10) Where the given line is outside the given curve and the curves reverse.

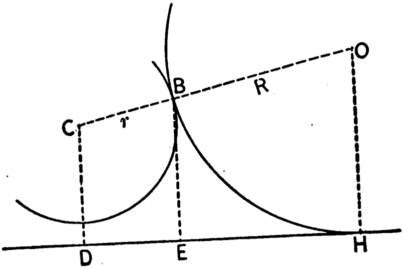

FIG. 10

$$\cos BCD = \frac{a - c}{r}$$

$$c = R - R \sin COH$$

$$R = \frac{c}{1 - \sin COH}$$

$$- \sin COH = \cos BCD = \frac{a - c}{r}$$

$$R = \frac{c}{1 + \frac{a - c}{r}} = \frac{cr}{r + a - c}$$

CASE 3. (Fig. 11) When the given line cuts the given curve. As shown in the figure, there are four sections.

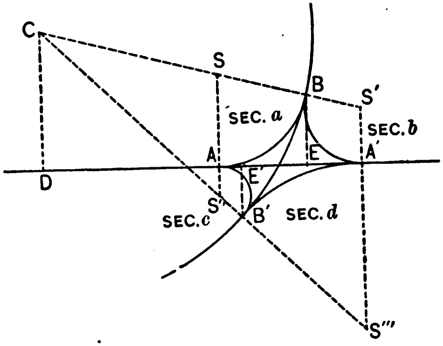

FIG. 11

Figure $CD = a$, $DE = b$, and $EB = c$.

Section *a*. $$\text{vers } BCD = \frac{r - a + c}{r}$$

$$R = \frac{BE}{\text{vers } BSA}$$

$$BSA = BCD$$

$$R = \frac{cr}{r - a + c}$$

$$DA = (r - R) \sin BCD$$

Section *b*. $$\cos. BCD = \frac{a - c}{r}$$

$$c = R + R \cos S'BE$$

$$S'BE = BCD$$

$$R = \frac{cr}{r + a - c}$$

$$DA' = (r + R) \sin BCD$$

Section *c*. $$\cos B'CD = \frac{a + c}{r}$$

$$c = R\,(1 + \cos B'SA)$$

$$180^\circ - B'S''A = B'CD$$

$$R = \frac{cr}{r + a + c}$$

Section *d*. $$\text{vers } B'CD = \frac{r - a - c}{r}$$

$$R = \frac{c}{\text{vers } B'S'''A'}$$

$$B'S'''A' = B'CD$$

$$R = \frac{cr}{r - a - c}$$

13. (Fig. 12) To pass a circular curve of a given radius, R, tangent to a given line, AD, and tangent to another given curve whose radius is r and whose center C is fixed in relation to the given line.

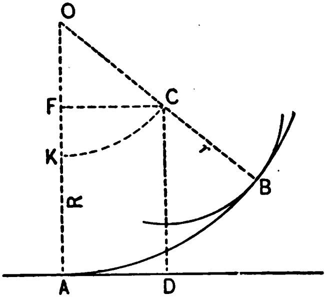

FIG. 12

CASE 1. Where the given line is outside the fixed curve and the curves do not reverse.

Figure the ordinate $CD = a$. $OC = R - r$.

Swing the curve of radius $R - r$ to intersect OA at K.

$$FK = a - r$$

$$\text{vers } AOB = \frac{a - r}{R - r}$$

$$AD = (r - R) \sin AOB$$

CASE 2. (Fig. 13) Where the given line is outside the fixed curve and the curves reverse.

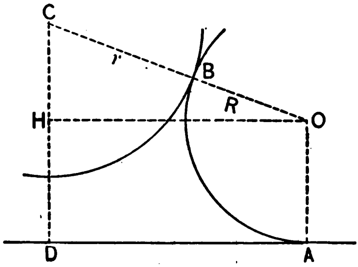

FIG. 13

Figure $CD = a$

$$OC = R + r \quad HC = a - R$$

$$\cos DCO = \frac{a - r}{R + r}$$

$$AD = (R + r) \sin DCO$$

$$AOB = 180^\circ - DCO$$

CASE 3. (Fig. 14) Where the given line cuts the fixed curve. Four sections are shown.

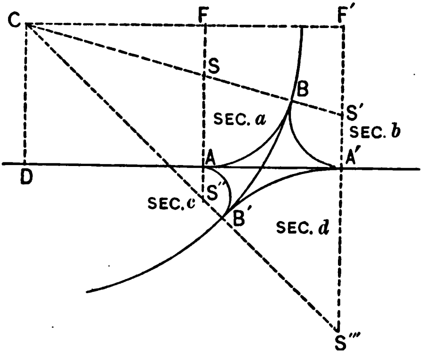

FIG. 14

Figure $DC = a$

Section *a*. $\cos BSA = \sin FCS$

$$\sin FCS = \frac{FS}{CS} = \frac{a - R}{r - R}$$

$$AD = (r - R) \cos FCS$$

Section *b*. $\cos BS'A' = \sin F'CS' = \dfrac{a - R}{r + R}$

$$A'D = (r + R) \cos F'CS'$$

Section *c*. $\cos B'S''A = \sin FCS'' = \dfrac{a + R}{r - R}$

$$AD = (r - R) \cos FCS''$$

Section *d*. $\cos B'S'''A' = \sin F'CS''' = \dfrac{a + R}{r + R}$

$$A'D = (r + R) \cos F'CS'''$$

14. (Fig. 15) To pass a circular curve through a given point, *B*, and tangent to a given curve at a given point, *A*.

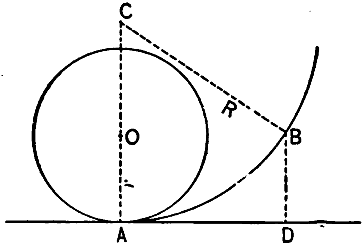

FIG. 15

Figure the co-ordinates *AD* and *DB* to a line drawn tangent through

A. From (17) $R = \frac{AD^2}{2\,BD} + \frac{BD}{2}$

15. (Fig. 16) To pass a circular curve tangent to a given circle and through two given points.

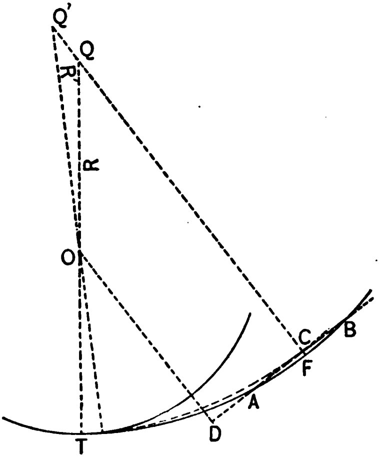

FIG. 16

Let O be the center of the given circle with its radius $OT = r$, and A and B be the two points and C the point midway between A and B on a line drawn through them.

Produce BA to D and draw OD at right angles with AB. Figure $OD = a$ and $DC = b$.

From problem 11, Case 1, the radius, R', of a curve tangent to the given curve, and tangent to AB at C, is

$$R^1 = \frac{b^2}{2\,(a - r)} + \frac{a + r}{2}$$

Assuming that the required radius, R, will not vary much in length from the radius R^1 we can write the middle ordinate $CF = \frac{\overline{AC}^2}{2R^1} = m$.

Using problem 11 again we may write

$$R = \frac{b^2}{2\,(a - r + m)} + \frac{a + r + m}{2}$$

With this value of R figure the value of m again; if there is no variation from the first value of m obtained, then R is the required radius; if there is a variation in it, then figure R again.

16. (Fig. 17) To pass a circular curve tangent to a given line, tangent to a given curve, and through a given point.

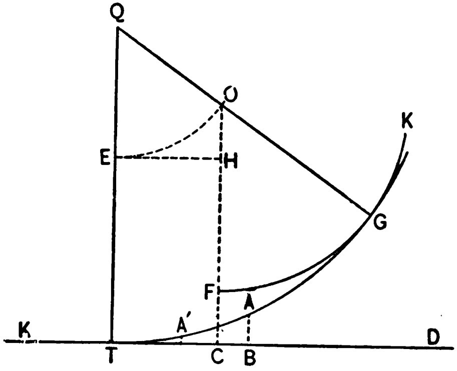

FIG. 17

CASE I. — When the line is outside the given curve. Let O be the center of the given curve, whose radius $OF = r$, KD be the given line, and A be the given point.

Figure $AB = a$, $CF = OC - r = b$, and $CB = c$.

Let $TB = x$

$$OQ = \mathrm{R} - r$$
$$EH = TC = x - c$$
$$OF = b$$

From (17)
$$R = \frac{x^2}{2\,a} + \frac{a}{2}$$

$$\text{and } R - r = \frac{(x - c)^2}{2\,b} + \frac{b}{2}$$

Substituting the value of R of the first equation in the second, and reducing

$$x^2 + \frac{2\,ac}{b - a}\,x = \frac{ab\,(2\,r - a + b) - ac^2}{b - a}$$

from which x may be found. R is found from the first equation and

$$\sin O = \sin Q = \frac{x - c}{R - r}$$

Being a quadratic equation two values of x are obtained. The minus value is for a curve tangent to the given curve on the side toward K.

If the point is at A' instead of at A the equation becomes

$$x^2 - \frac{2\,ac}{b - a}\,x = \frac{ab\,(2\,r - a + b) - ac^2}{b - a}$$

CASE 2. (Fig. 18) When the line cuts the given curve.

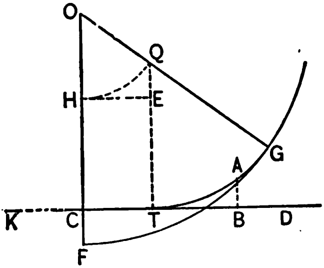

FIG. 18

Figure $AB = a$, $CF = b$, and $CB = c$.
Let $TB = x$

$$OQ = r - R$$
$$EH = c - x$$
$$EQ = b$$
$$R = \frac{x^2}{2a} + \frac{a}{2}$$
$$r - R = \frac{(c-x)^2}{2b} + \frac{b}{2}$$
$$x^2 - \frac{2ac}{b+a}x = \frac{ab(2r-a-b) - ac^2}{b+a}$$

17. (Fig. 19) To pass a circular curve tangent to a given line and tangent to two given curves.

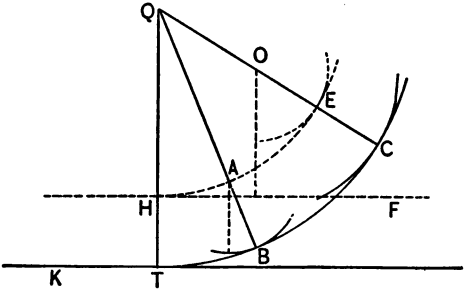

FIG. 19

Let AB be the radius of the first given curve and OC the radius of the second.

Draw HF parallel with KT and distant from it by the length of the radius AB, and with the center O draw the circle with the radius $OE = OC - AB$.

From problem 16 figure a curve tangent to HF through A and tangent to the circle whose radius is OE.

$$R = QT = QC = QE + AB$$

18. (Fig. 20) To pass a line tangent to two given circles.

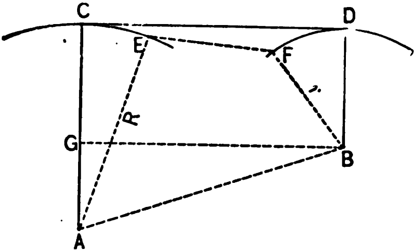

FIG. 20

Given the radii R and r, the distance EF, and the angles FEA and EFB.

Figure AB and the angles EAB and FBA.

$$AG = R - r$$

$$\sin ABG = \frac{AG}{AB}$$

$$CD = GB = AB \cos. ABG$$

$$CAE = 90^\circ - ABG - EAB$$

$$DBF = 90^\circ + ABG - FBA$$

19. (Fig. 21) To pass a circular curve of a given radius tangent to two other given curves.

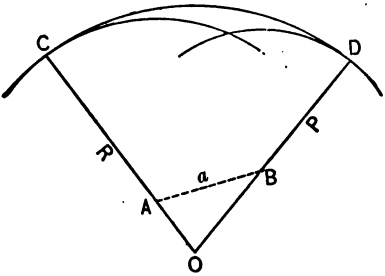

FIG. 21

Let R and P be the radii of the two curves whose position is fixed.

Figure the distance AB between centers.

Let s be the radius of the third curve.

$$OA = s - R$$

$$OB = s - P$$

From the three sides given figure the angles in the triangle OAB.

If the curve of radius P were convex to the other curves the sides of the triangle would be $s + R$, $s + P$, and AB. If it were concave to the curve of radius R and convex to the curve of radius P the sides of the triangle would be $s - R$, $s + P$, and AB.

20. (Fig. 22) To pass a circular curve tangent to a given curve at a given point on its circumference and tangent to another given curve.

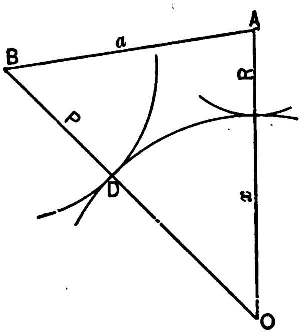

FIG. 22

Let D be the point on the curve of radius P, let R be the radius of the other fixed curve, and $a = AB$, the distance between centers and let x be the radius of the required curve.

Figure the angle $B = 180° - ABD$.

CASE 1. (Fig. 22) In triangle AOB

$$\overline{AO^2} = \overline{AB^2} + \overline{OB^2} - 2\ AB \times OB \times \cos ABO$$

or

$$(x - R)^2 = a^2 + (x - P)^2 - 2a\,(x - P)\cos B$$

Reducing, we have

$$x = \frac{(a + P - R)\,(a - P + R)}{2\,(P - R + a\cos B)} + P$$

CASE 2. (Fig. 22) If the third curve were convex to the two fixed curves, by the same method,

$$x = \frac{(a + P - R)\,(a - P + R)}{2\,(R - P + a\cos B)} - P$$

CASE 3. (Fig. 23) If the third curve were concave to one curve at the given point *D* and convex to the other curve, by the same method

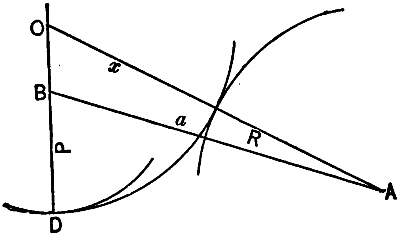

FIG. 23

$$x = \frac{(a + R + P)\,(a - R - P)}{2\,(R + P + a \cos B)} + P$$

CASE 4. (Fig. 24) If the third curve were convex to one curve at the given point *D*, and concave to the other curve, the formula is

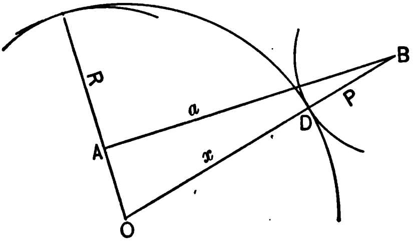

FIG. 24

$$x = \frac{(a + R + P)\ \ (R + P - a)}{2\,(R + P - a \cos B)} - P$$

When B is over 90°, remember that

$$\cos\,(180° - B) = -\cos B$$

21. (Fig. 25) Given the radii R and P, the angles O and Q, and the deflection angle $D = O + Q$, to find the common tangents CD and DE.

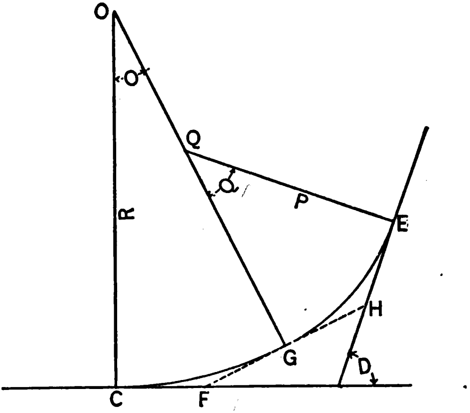

FIG. 25

In laying out the curves on the ground the tangents $CF = FG$ and $GH = HE$ are almost always required, hence it is better to figure them.

In the triangle FHD, having the angles and $FH = FG + GH$, figure FD and DH.

$$CD = CF + FD$$
$$DE = DH + HE$$

In a similar manner, by combining the common tangents with the tangents of a third curve the common tangents of these curves may be found.

22. Given the angles O and Q, the deflection angle D, the radius R, and the common tangent CD, to find the radius P and the common tangent DE.

$$\Delta FDH \begin{cases} FD = CD - R \tan \tfrac{1}{2} O \\ FH = \dfrac{FD}{\sin Q} \sin D \end{cases}$$

$$DH = \frac{FD}{\sin Q} \sin O$$

$$GH = FH - R \tan \tfrac{1}{2} D$$
$$P = GH \cot \tfrac{1}{2} Q$$
$$DE = DH + GH$$

23. (Fig. 26) Given the angles O, Q, and D, the radius R, and the common tangent DE, to find the radius P and the common tangent CD.

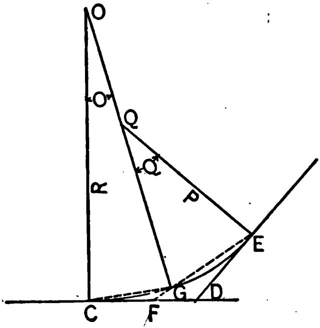

FIG. 26

$$\Delta FED \begin{cases} FD = \dfrac{ED}{\sin(D - \frac{1}{2}Q)} \sin \frac{1}{2}Q \\ FE = \dfrac{ED}{\sin(D - \frac{1}{2}Q)} \sin D \end{cases}$$

$$CG = 2\,R \sin \tfrac{1}{2}\,O$$

$$\Delta CFG \quad FG = CG \sin \tfrac{1}{2}\,O \operatorname{cosec}(D - \tfrac{1}{2}\,Q)$$

$$P = \frac{FE - FG}{2 \sin \frac{1}{2}\,Q}$$

To find CD proceed as in problem 21, or

$$CD = FD + 2\,R \sin \tfrac{1}{2}\,O \operatorname{cosec}(D - \tfrac{1}{2}\,Q) \sin \tfrac{1}{2}D$$

24. (Same figure.) Given the angles O, Q, and D and the common tangents CD and DE, to find the radii R and P.

Find FD and FE as in problem 23.

$$CF = CD - FD$$

$$\Delta CFG \begin{cases} CG = \dfrac{CF}{\sin \frac{1}{2}\,D} \sin(D - \frac{1}{2}\,Q) \\ FG = \dfrac{CF}{\sin \frac{1}{2}\,D} \sin \frac{1}{2}\,O \end{cases}$$

$$EG = FE - FG$$

$$R = \frac{CG}{2} \operatorname{cosec} \tfrac{1}{2}\,O$$

$$P = \frac{EG}{2} \operatorname{cosec} \tfrac{1}{2}\,Q$$

25. (Fig. 27) Given the radii R and P, the angle D, and the common tangent DE, to find the angles O and Q and the common tangent CD.

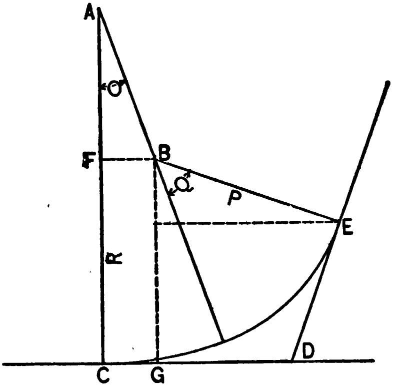

FIG. 27

$$FC = BG = P \cos D + DE \sin D$$
$$DG = \mathrm{P} \sin D - DE \cos D$$
$$AB = R - P$$
$$AF = R - FC$$

$$\cos O = \frac{AF}{AB}$$

$$Q = D - O$$

$$CD = DG + FB = DG + AB \sin O$$

26. (Fig. 28) Given the radii R and P, the angle D, and the common tangent CD, to find the angles O and Q and the common tangent DE.

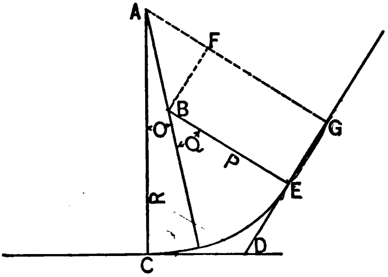

FIG. 28

$$
\begin{aligned}
AG &= CD \sin D + R \cos D \\
GD &= R \sin D - CD \cos D \\
AF &= AG - P \\
AB &= R - P
\end{aligned}
$$

$$\cos Q = \frac{AF}{AB} .$$

$$
\begin{aligned}
O &= D - Q \\
DE &= GD - EG = GD - AB \sin Q
\end{aligned}
$$

27. (Fig. 29) Given the radius *R*, the common tangents *CD* and *DE*, and the angle *D*, to find the angles *O* and *Q* and the radius *P*.

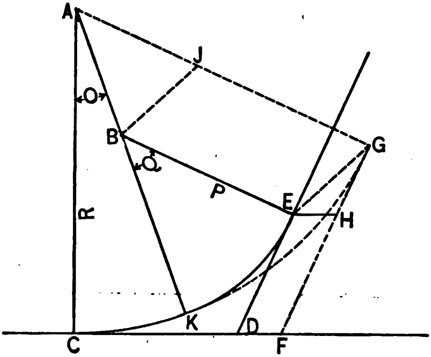

FIG. 29

$$CF = FG = R \tan \tfrac{1}{2} D$$
$$EH = DF = CF - CD$$
$$HG = FG - DE$$

In Δ *EGH*, having *EH*, *HG*, and $EHG = 180° - D$, figure $EGH = \frac{1}{2} Q$ and $EG = BJ$.

$$R - P = \frac{BJ}{2 \sin \frac{1}{2} Q}$$
$$O = D - Q$$

28. (Fig. 30) Given the radius P, the common tangents CD and DE, and the angle D, to find the radius R and the angles O and Q.

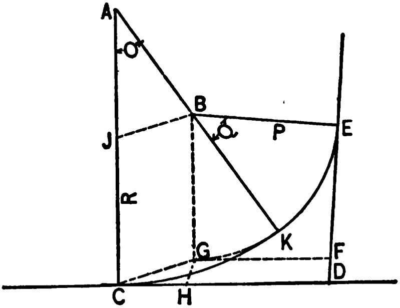

FIG. 30

$$GF = FE = P \tan \tfrac{1}{2} D$$
$$GH = FD = DE - FE$$
$$CH = CD - GF$$

In $\Delta\, CGH$, having GH, CH, and $CHG = 180° - D$, figure $GCH = \tfrac{1}{2} O$ and $CG = BJ$.

$$R - P = \frac{BJ}{2 \sin \tfrac{1}{2} O}$$
$$Q = D - O$$

29. (Fig. 31) Given the common tangents $CD = DE$, the angle D, the radii $R = R^1$, and the distance GD, to find the radius P and the angles O and Q.

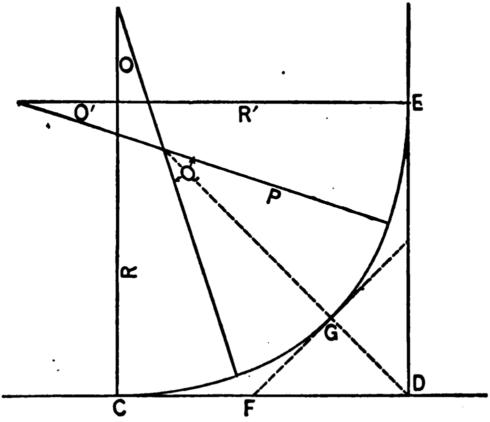

FIG. 31

$$
\begin{aligned}
FGD &= 90^\circ \\
GFD &= \tfrac{1}{2}\, D \\
FG &= GD \cot. \tfrac{1}{2}\, D \\
CF &= CD - GD \operatorname{cosec} \tfrac{1}{2}\, D
\end{aligned}
$$

Having the common tangents CF and FG, the angles GFD and the radius R, figure by problem 27 the radius P and the angles O and $\frac{1}{2}\, Q$.

30. (Same figure.) Given the radii $R = R^1$, the angles $O = O^1$, Q and D, and the distance DG, to find the radius P and the common tangents $CD = DE$.

$$
\begin{aligned}
GFD &= \tfrac{1}{2}\, D \\
FD &= GD \operatorname{cosec} \tfrac{1}{2}\, D \\
FG &= GD \cot \tfrac{1}{2}\, D
\end{aligned}
$$

Having the angles O, $\frac{1}{2}\, Q$, and GFD, the radius R, and the common tangent FG, figure by problem 23 the radius P and CF.

$$CF + FD = \text{common tangent } CD$$

31. (Fig. 32) Given the radii $R = R^1$, and P, and the angles $O = O^1$, Q and D, to find the common tangents $CD = DE$.

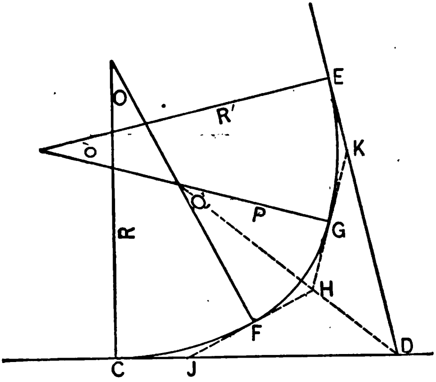

FIG. 32

$$
\begin{aligned}
CJ &= JF = GK = KE = R \tan \tfrac{1}{2} O \\
FH &= HG = P \tan \tfrac{1}{2} Q \\
JH &= JF + FH \\
HJD &= O \\
HDJ &= 90^\circ - \tfrac{1}{2} D
\end{aligned}
$$

$$
\Delta FHDJD = \frac{JH}{\cos \frac{1}{2} D} \cos \left(\tfrac{1}{2} D - O\right)
$$

32. (Fig. 32) Given the common tangents $CD = DE$, the radii $R = R^1$, and the angles $O = O^1$, Q and D, to find the radius P.

$$
\begin{aligned}
CJ &= R \tan \tfrac{1}{2} O \\
DJ &= CD - CJ \\
HJD &= O \\
HDJ &= 90^\circ - \tfrac{1}{2} D \\
DHJ &= 180^\circ - O - (90^\circ - \tfrac{1}{2} D) = 90^\circ + \tfrac{1}{2} D - O \\
JH &= \frac{JD \cos \frac{1}{2} D}{\cos \left(\frac{1}{2} D - O\right)} \\
FH &= JH - CJ \\
P &= FH \cot \tfrac{1}{2} Q
\end{aligned}
$$

33. (Fig. 33) Given the common tangents *ED* and *DF*, the radii *R*, *P*, and *S*, and the angle *D*, to find the angles *O*, *Q*, and *C*.

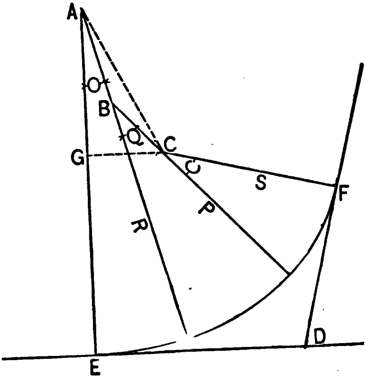

FIG. 33

$$AG = R \quad - FD \sin D - S \cos D$$
$$GC = ED + FD \cos D - S \sin D$$

$$\tan CAE = \frac{GC}{AG}$$

$$AC = AG \sec CAE$$

Having AC, $AB = R - P$ and $BC = P - S$, figure the angles in ΔABC.

$$O = CAE - BAC$$
$$Q = 180^\circ \quad - ABC$$
$$C = D - (O + Q)$$

34. (Fig. 34) Given the common tangents ED and DF, the angles D and Q, and the radii P and S, to find the radius R and the angles O and C.

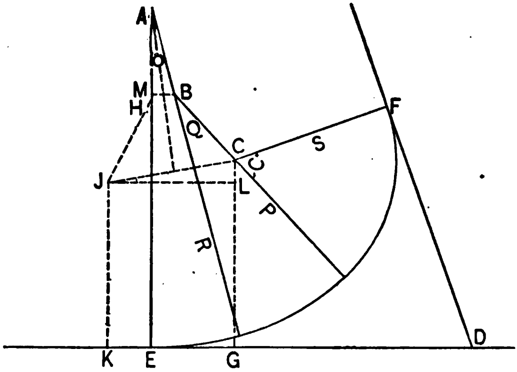

FIG. 34

$$EG = ED - FD \cos D - S \sin D$$
$$GC = FD \sin D - S \cos D$$

Let $EH = P$, $HJ = P - S$ and $EHJ = Q$.

$$JL = EG + (P - S) \sin Q$$
$$CL = GC - JK = GC - (P - (P - S) \cos Q)$$

In the polygon $JHABCJ$, since $JH = BC$, $AH = AB$, and $EHJ = Q$, a line bisecting O must be at right angles with JC. As JL is at right angles with AE, $CJL = \frac{1}{2} O$.

$$\tan \tfrac{1}{2} O = \frac{CL}{JL}$$

$$C = D - (O + Q)$$
$$BM = EG - (P - S) \sin (O + Q)$$
$$R = P + BM \operatorname{cosec} O$$

CLEARANCE

The term clearance means that much distance between the rail and any obstruction, such as a manhole or a building, as may be required. Problem 16 is a good illustration of a clearance case in which A is the center of a manhole cover, KD the center of main line, and GK the center line of another curved track coming into the main track, where it is desired to keep the manhole cover in the middle of the track. Problem 11 affords another case where C is the corner of a building, AD the center line of track, and BC the amount of clearance from the corner of the building to the center line of a branch-off track. CB includes the overhang of the car plus half the gage, plus the amount of clearance from the side of the car to the corner. Problem 10 can be used where it is required to run a branch-off clearing two manhole covers, D and C being the centers of the manholes. Problem 15 is a similar case.

Problem 17 is a case where the branch-off is to clear a point at A and connect with the curve whose center is O or where the points A and O are to be cleared by the distances AB and OC respectively. This last case illustrates the method of first figuring the curve to run through the points to be cleared and then adding the amount of clearance on to the radius of the curve found.

Problems 35 and 36 are two more clearance problems worked out approximately.

35. (Fig. 35) To pass a curve tangent to two given curves and approximately through a given point.

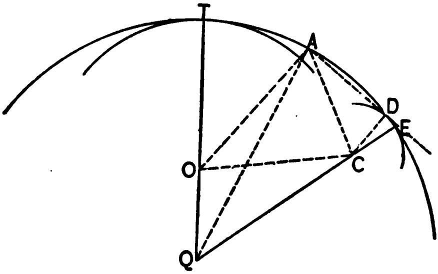

FIG. 35

Let O and C be the centers of the given curves with the radii OT and CE respectively.

Figure the line AD tangent to the curve at D.

Using problem 15 figure the radius, R, of a curve passing through A and D and tangent to the curve whose center is O.

In the triangle OCQ, calling $OQ = R - OT$ and $QC = R - CE$, and figuring OC, we may find the angles and thus establish the point T and E.

If the difference $\frac{AE^2}{2R} - \frac{AD^2}{2R}$ is too much to be neglected, add this difference to R for a new value of R and re-figure the angles.

36. (Fig. 36) To pass a curve tangent to two given curves and approximately tangent to a third given curve.

Let O and C be the centers of the first two curves and A the center of the third.

With O for a center, and radius $OB = OT - AE$, draw the curve BG, and with C for a center, and radius $CD = CF - AE$, draw curve DH. The problem now reduces itself to problem 35 with the center of the circle, A, as the given point.

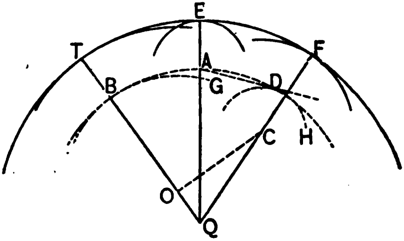

FIG. 36

The required radius is $R + AE$.

If the radius of the middle curve AE is longer than the outer curve CF, draw the curve at C with radius $AE - CF$, and figure the large curve to run tangent to the side of the curve nearest Q.

REVERSED CURVES

37. (Fig. 37) Given the distance a between two parallel lines and the common radius R, to find the angle O, and the distance AD.

$$OC = R - \tfrac{1}{2} a$$

$$\cos O = \frac{OC}{R}$$

$$AD = 2 R \sin O$$

38. (Fig. 37) Given a and AD, to find R and O.

$$\tan \tfrac{1}{2} O = \frac{a}{AD}$$

$$R = \tfrac{1}{2} AD \operatorname{cosec} O$$

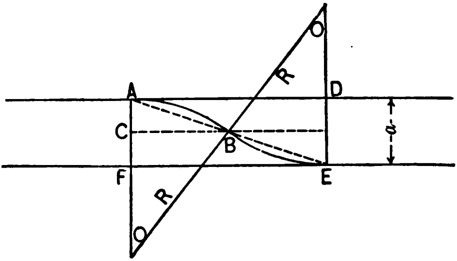

FIG. 37

39. (Fig. 37) Given a and AE, to find R, O, and AD.

$$\sin \tfrac{1}{2} O = \frac{a}{AE}$$

$$R = \frac{AE}{4} \operatorname{cosec} \tfrac{1}{2} O$$

$$AD = AE \cos \tfrac{1}{2} O$$

40. (Fig. 38) Given a, AB, and R, to find O and radius P.

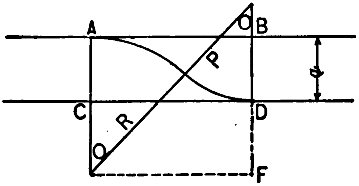

FIG. 38

$$\tan \tfrac{1}{2} O = \frac{a}{AB}$$

$$P = AB \sec O - R$$

41. (Fig. 38) Given a, R, and P, to find AB and O.

$$OF = R + P - a$$

$$\cos O = \frac{OF}{R+P} \text{ or vers } O = \frac{a}{R+P}$$

$$AB = (R + P) \sin O$$

42. (Fig. 39) Given the common tangent AB, the angles $BAC = O$, and $ABD = Q$, to find the common radius R.

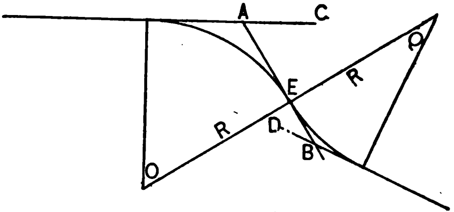

FIG. 39

$$R \tan \tfrac{1}{2} O = AE$$

$$R \tan \tfrac{1}{2} Q = EB$$

$$AB = AE + EB = R \tan \tfrac{1}{2} O + R \tan \tfrac{1}{2} Q$$

$$R = \frac{AB}{\tan \frac{1}{2} O + \tan \frac{1}{2} Q} = \frac{AB \cos \frac{1}{2} O \cos \frac{1}{2} Q}{\sin \frac{1}{2} (O + Q)}$$

43. (Fig. 40) Given the distance, *AB*, from the *P. C.* to the *P. T.* of a reversed curve and the angles *A* and *B*, to find the common radius *R* and the angles *O* and *Q*.

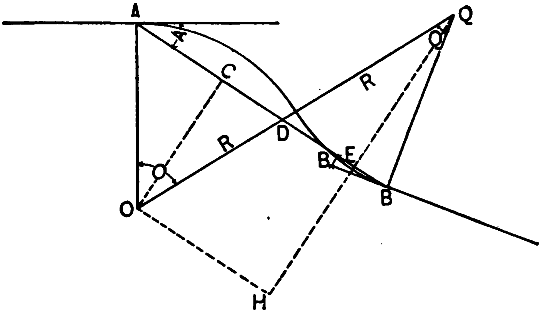

FIG. 40

$$AC = R \sin A$$
$$OC = R \cos A$$
$$EB = R \sin B$$
$$QE = R \cos B$$
$$QH = OC + QE$$
$$D = ADO = QOH = QDB$$
$$QH = 2\,R \sin D = R \cos A + R \cos B$$
$$\sin D = \tfrac{1}{2}(\cos A + \cos B) = \cos \tfrac{1}{2}(A + B) \cos \tfrac{1}{2}(A - B)$$
$$AB = AC + CE + EB = R \sin A + 2\,R \cos D + R \sin B$$
$$R = \frac{AB}{\sin A + \sin B + 2 \cos D}$$
$$O = A + 90^\circ - D$$
$$Q = B + 90^\circ - D$$

44. (Fig. 41) Given the angles *A* and *B*, the radii *R* and *P*, and the distance *AB* from the *P. C.* at *A* of a reversed curve to any point *B* on tangent line *BD*, to find the distance *BD* and angles *O* and *Q*.

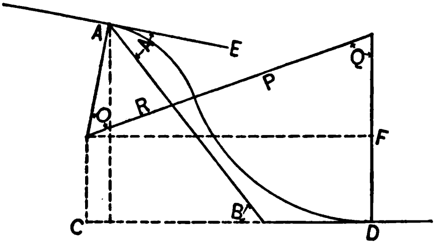

FIG. 41

$$OC = FD = AB \sin B - R \cos (B - A)$$
$$CB = AB \cos B + R \sin (B - A)$$
$$QF = P - FD$$
$$\sin QOF = \frac{QF}{R + P}$$
$$CD = OF = (R + P) \cos QOF$$
$$BD = CD - CB$$
$$Q = 90^\circ - QOF$$
$$O = Q + A - B$$

45. (Fig. 42) Given *AB*, from *P. C.* to *P. T.* of a reversed curve, the angles *A* and *B*, and the radius *R*, to find the radius *P* and the angles *O* and *Q*.

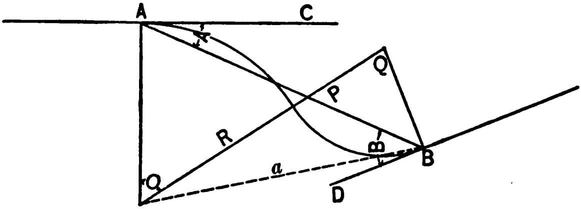

FIG. 42

In Δ *ABO*, figure $OB = a$ and *ABO*.

$$T = QBO = 90^\circ - B + ABO$$

In Δ *OQB*, $\overline{QO}^2 = \overline{QB}^2 + \overline{OB}^2 - 2\ QB \times OB \times \cos T$, or $(R + P)^2 = P^2 + a^2 - 2\,a\,P \cos T$.

$$P = \frac{(a + R)\ (a - R)}{2\ (R + a \cos T)}$$

In Δ *OQB*, $\sin Q = \dfrac{\sin T}{R + P}\, a$

$$O = Q + A - B$$

TONGUE SWITCH SPECIAL WORK

Throughout this subject g represents the gage, F the frog-angle and frog-point, R_o, P_o, etc., the radii to the center of the track, and $R_o + \frac{g}{2} = R$, $R_o - \frac{g}{2} = r$, etc.

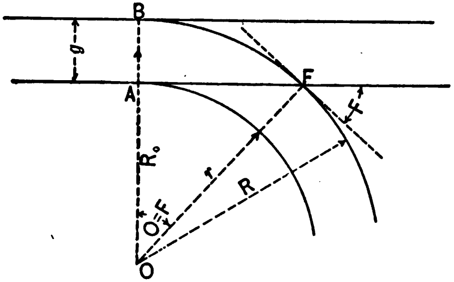

FIG. 43

46. (Fig. 43) Given g, R_o, to find F and frog distance $AF = d$.

$$\cos F = \frac{r}{R} \text{ or}$$

$$\text{vers } F = \frac{g}{R} \text{ or}$$

$$\text{Sin } \tfrac{1}{2} F = \sqrt{\frac{g}{2R}}$$

$$d = R \sin F \text{ or}$$

$$d = \sqrt{(R_o + \tfrac{1}{2} g)^2 - (R_o - \tfrac{1}{2} g)^2}$$

$$= \sqrt{2 R_o g}$$

$$\text{chord } BF = 2 R \sin \tfrac{1}{2} F$$

$$= \sqrt{2 R g}$$

Having found d, the arc BF may be found by using slide rule (5).

This form, known as a plain branch-off, is restricted in actual work to a small number of cases. Manufacturers of special work have certain standard patterns of tongue switches made to certain radii. The practice is to compound these standard switches with the branch-off curve; for instance, a 95-ft. branch-off might be combined with a 100-ft. radius switch.

Branch-offs and switches are designated right-hand and left-hand according to direction of branch-off in leaving the straight track.

The proper place for the tongue switch is on the inside of the curve (at A in the figure); for when the tongue is on the outside, the car in taking the switch subjects the tongue to two forces: the weight of the car and the centrifugal force developed in the car in passing over the curve. When the tongue is on the inside of the curve, the truck-wheels ride the tongue only when passing on the straight track, and when passing on the curve the centrifugal force is mostly expended against the mate, which, being a solid piece, is more capable of withstanding it.

The tongue-point is always back of the theoretical switch-point owing to the impracticability of bringing the tongue to an actual point. Also, for the reason that proper contact with the main rail of the switch must be made, the curve of the tongue is made flatter than the radius of the switch.

When a branch-off runs from one gage on the straight track to a slightly wider gage on the curve, the difference is taken up in the mate, its spread being made that much less.

47. (Fig. 44) Given g, R, and P and $BOC = O$, to find F and $AF = d$.

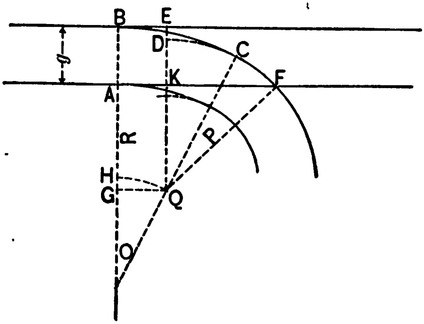

FIG. 44

Run the curve CD of radius P to D when QD is perpendicular with AF.

$$c = DE = HG = R \text{ vers } O - P \text{ vers } O = (R - P) \text{ vers } O$$

$$b = AK = GQ = R \sin O - P \sin O = (R - P) \sin O$$

$$DK = g - c$$

$$\text{vers } F = \frac{DK}{P} \quad \text{or} \quad \cos F = \frac{P - DK}{P}$$

$$CQF = F - O$$

$$d = AF = b + P \sin F \quad \text{or}$$

$$d = b + \sqrt{(P_o + \tfrac{1}{2}\, g)^2 - (P_o - \tfrac{1}{2}\, g + c)^2} = b + \sqrt{(2\, P_o + c)\, (g - c)}$$

$$\tan \tfrac{1}{2} F = \frac{g - c}{\sqrt{(2\, P_o + c)\, (g - c)}} = \sqrt{\frac{g - c}{2\, P_o + c}}$$

These last two equations are valuable for slide-rule checks. Also, almost always, c and b may be found accurately on the slide rule.

48. (Fig. 45) Given g, R, O, and $AF = d$, to find F and P.

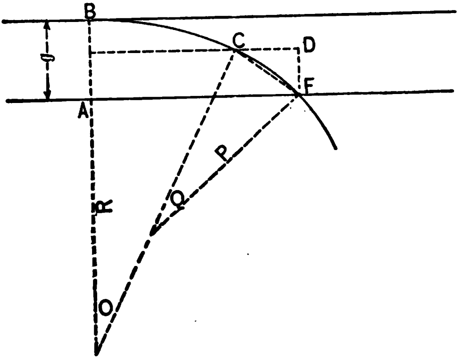

FIG. 45

$$CD = d - R \sin O$$
$$DF = g - R \operatorname{vers} O$$
$$\tan DCF = \frac{DF}{CD}$$
$$\tfrac{1}{2} Q = DCF - O$$
$$F = O + Q$$
$$P = \tfrac{1}{2} CF \operatorname{cosec} \tfrac{1}{2} Q = \tfrac{1}{2} CD \sec DCF \operatorname{cosec} \tfrac{1}{2} Q$$

49. (Fig. 46) Given g, R, and P, and F, to find O, Q, and $AF = d$.

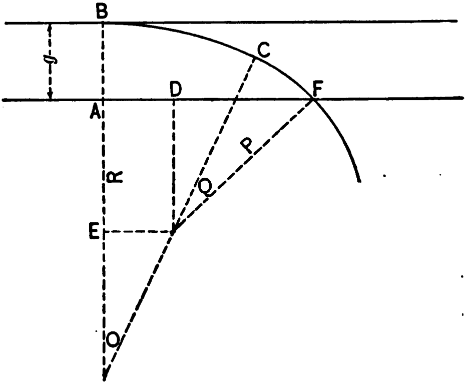

FIG. 46

$$DF = P \sin F$$
$$QD = P \cos F$$

$$EO = r - QD$$
$$\cos O = \frac{EO}{R - P}$$
$$Q = F - O$$
$$d = AF = (R - P) \sin O + DF$$

50. (Fig. 47) Given g, R, and O, to find $AF = d$ and CF.

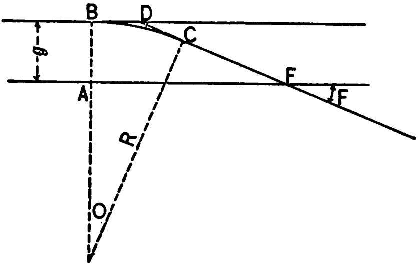

FIG. 47

$$F = O$$
$$BD = DC = R \tan \tfrac{1}{2} O$$
$$d = AF = BD + g \cot O$$
$$CF = g \operatorname{cosec} O - DC$$

51. (Fig. 48) Given g, radii R, P, and S, angles O and Q, to find F and AF.

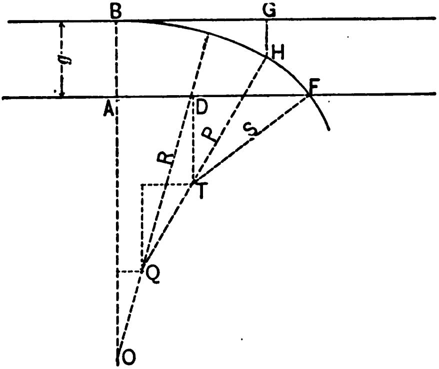

FIG. 48

$$\begin{aligned} AD &= (R - P)\sin O + (P - S)\sin(O + Q) \\ TD &= r - (R - P)\cos O - (P - S)\cos(O + Q) \\ \cos F &= \frac{TD}{S} \\ AF &= AD + S\sin F \\ T &= F - (O + Q) \end{aligned}$$

This solution is applicable to any number of compounds between B and F as in easement curves. In the latter case the co-ordinates $BG = y$ and $GH = x$ would be given, then

$$\begin{aligned} AD &= y - S\sin(O + Q) \\ TD &= S + x - S\ \text{vers}\ (O + Q) \end{aligned}$$

52. (Fig. 49) Given g, R, and P, to find F, QOF, and OQF. Figure the angles from the three sides given:

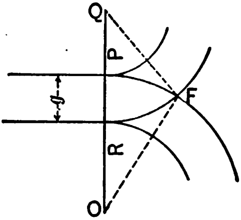

FIG. 49

$$OQ = R_o + OQ = R_o + P_o,\ OF = R \text{ and } QF = P$$

$$F = O + Q$$

If $P = R$, then $OQF = QOF = \frac{1}{2} F$.

$$\cos \tfrac{1}{2} F = \frac{R_o}{R}$$

53. (Fig. 50) Given g, R_o, P_o, and $AB = a$, to find F, O, and Q.

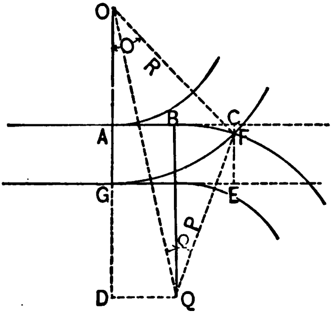

FIG. 50

$$\tan DOQ = \frac{a}{R_o + P_o}$$

$$OQ = (R_o + P_o) \sec DOQ$$

Having three sides given in OQF, figure the angles.

$$F = 180^\circ - OFQ$$
$$O = QOF + DOQ$$
$$Q = OQF - DOQ$$

54. (Fig. 51) Given g, R_o, P_o, S_o, the angle C, and a, to find F **and** the arcs DF and BF.

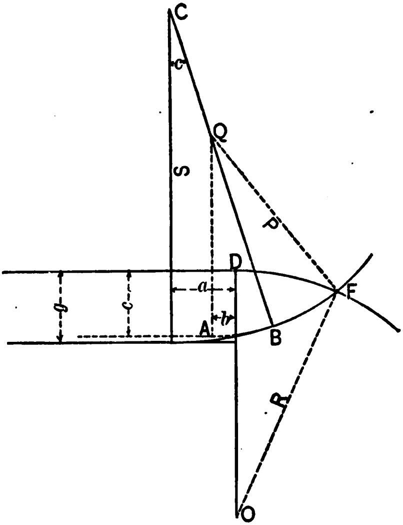

Fig. 51

$$b = a - (S_o - P_o)\sin C$$
$$c = g - (S_o - P_o)\operatorname{vers} C$$

Using c in place of g and b in place of a, figure as in Problem 53, finding the frog-angle F and arcs DF and AF. Subtract the arc AB from AF for the arc BF.

Note the similarity with problem 47 in the preliminary figuring.

If the arc DF is also compounded make a similar difference in Band C in the same manner as given above.

55. (Fig. 52) Given g, R_0 and P_0, O and a, to find F, FE, and arc DF.

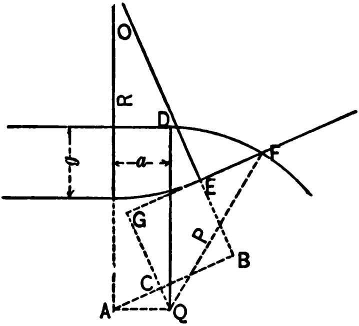

FIG. 52

$$\begin{aligned}
OA &= R_0 + P_0\\
OB &= OA \cos O\\
AB &= OA \sin O\\
AQ &= a \text{ and } QAB = O\\
AC &= a \cos O\\
QC &= a \sin O\\
QG &= OB + QC - R
\end{aligned}$$

$$\cos F = \cos GQF = \frac{QG}{P}$$

$$\begin{aligned}
Q &= F - O\\
GF &= P \sin F\\
EF &= GF + AC - AB
\end{aligned}$$

56. (Fig. 53) Given g, $Ro = Po$ and a, to find F, O, and Q.

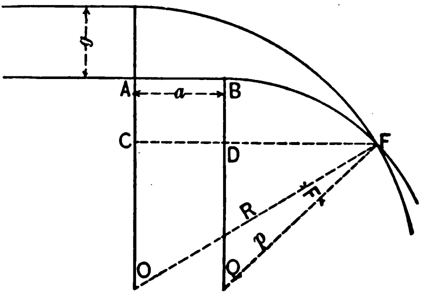

FIG. 53

Let $DF = x$.

$$\overline{OC}^2 = \overline{QD}^2 = R^2 - (a + x)^2 = p^2 - x^2$$

$$x = \frac{R^2 - p^2 - a^2}{2a} = \frac{(R + p)\ (R - p) - a^2}{2\ a}$$

$$= \frac{gR_o}{a} - \frac{a}{2}$$

$$\sin Q = \frac{x}{p} \text{ and } \sin O = \frac{a + x}{R}$$

$$F = O - Q$$

57. (Fig. 54) Given g, R_o, P_o, and a, to find F, O, and Q.

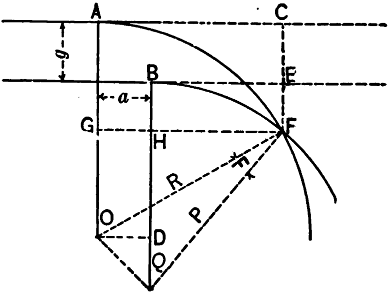

FIG. 54

$$QD = P_o - R_o$$

$$\tan OQD = \frac{a}{QD}$$

$$OQ = QD \sec OQD$$

In triangle OFQ, having three sides given, figure the angles.

$$\begin{aligned} F &= OFQ \\ O &= 180^\circ - QOF - OQD \\ Q &= OQF - OQD \end{aligned}$$

SPREADS

58. (Fig. 55) The spread of a frog is the distance AD or BC measured from points equidistant from the frog-point. They are used for sending into the shop for making up frogs.

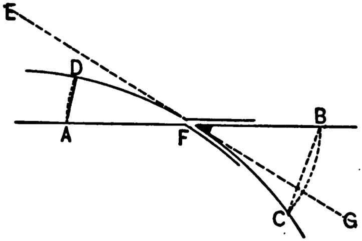

FIG. 55

$$\begin{aligned} F &= BFG = EFA \\ a &= AF = DF = BF = CF \end{aligned}$$

The distance "a" need not be the actual length of the frog-arm. Draw GE tangent to the curve at F; the deflection angles between the chords and tangent $EFD = GFC = d$.

$$\begin{aligned} AFD &= F - d \\ BFD &= F + d \\ AD &= 2a \sin \tfrac{1}{2}\, AFD \\ BC &= 2\, a \sin \tfrac{1}{2}\, BFC \end{aligned}$$

These may be figured on the slide rule for any length of a; but where the spread is always given for three feet the tables will be found more convenient.

In the deflection table the radius for each change of one minute for 1′ to 3° 00′ is given. In the spread table the angle is given for a change of $\frac{1}{16}$ of an inch in the spread.

EXAMPLE 1. $F = 37^\circ\ 25'$. $R = 47.645$.

37° 25′	
1 48 . .	*cor.* for 47.645
39 13 . .	2′ 0 $\frac{3}{16}$″ spread BC
35 37 . .	1′ 10″ spread AD

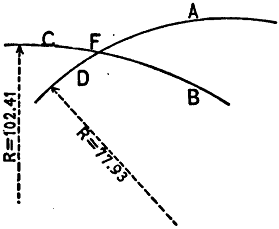

FIG. 56

EXAMPLE 2. (Fig. 56) cor. for 77.93 = 1° 06′
" " 102.41 = 0 50
Total cor. = 0 16

F = 18° 41′
cor. = 0 16
18 57 . . 11⅞″ spread *CD*
18 25 11½″ " *AB*

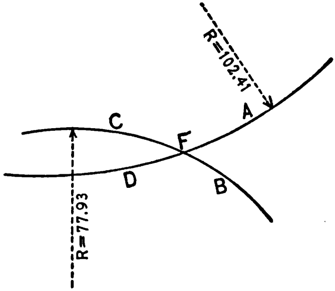

FIG. 57

EXAMPLE 3. (Fig. 57) cor. for 77.93 = 1° 06′
" " 102.41 = 0 50′
Total cor. = 1 56

F = 18° 41′
cor. = 1 56
20 37 . . 0′ 10½″ spread *CD*
16 45 . . 1′ 0⅞″ spread *AB*

CROSSINGS

59. (Fig. 58) Given the angle, $ACB = O$, between the center line of the straight track and the tangent to the center line of the curved track and the radius R, to find the rail lengths between frog-points and diagonal lengths.

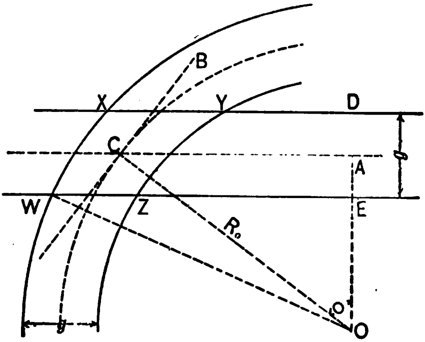

FIG. 58

$$Mo = OA = R_o \cos O$$
$$M = M_o + \tfrac{1}{2}\, g$$
$$m = M_o - \tfrac{1}{2}\, g$$
$$WE = \sqrt{R^2 - m^2} = \sqrt{(R + m)\,(R - m)}$$
$$ZE = \sqrt{(r + m)\,(r - m)}$$
$$XD = \sqrt{(R + M)\,(R - M)}$$
$$YD = \sqrt{(r + M)\,(r - M)}$$
$$XY = XD - YD$$
$$WZ = WE - ZE$$

$XW = \sqrt{g^2 + (WE - XD)^2}$, using "Table of Right-angled Triangles."

$$ZY = \sqrt{g^2 + (ZE - YD)^2}$$
$$WY = \sqrt{g^2 + (WE - YD)^2}$$
$$XZ = \sqrt{g^2 + (XD - ZE)^2}$$

In many cases the curves are so flat that there is no appreciable difference in length between the chord and arc; if, however, there is an appreciable difference it may be found by slide rule from (4).

For steam railroad crossings the frog-angles are of no value as the work is laid out in the shop from the rail lengths and diagonals; but where separate frogs are to be made, the frog-angles are found from

$$\sin WOE = \frac{WE}{R}, \text{ or } \tan WOE = \frac{WE}{m}, \text{ etc.}$$

The table, "Right-angled Triangles," referred to above, will be found convenient where the gage is standard or 4.71 feet. The table gives a change of each sixteenth of an inch in the hypothenuse from 4′ 8½″ to 10′ 0″, and a change of a quarter of an inch from

10′ 0″ to 20′ 7¾″. The large figures in the columns and the small figures in the column at the extreme left are to be taken together for the hypothenuse, and the small figures in the columns for the base, *e.g.*, Base = 6.234, gives hypothenuse = 7′ 9¾″.

Example: $R_o = 5729.655$, $ACB = 71^\circ\ 38'$.

$$\begin{array}{rlrlrl}
\log 5729.655 = & 3.7581285 & R_o = & 5729.655 & M_o = & 1805.397\\
\log\cos 71^\circ 38' = & \underline{9.4984442} & & \underline{2.355} & & \underline{2.355}\\
M_o = 1805.397 & 3.2565727 & R = & 5732.010 & M = & 1807.752\\
 & & r = & 5727.300 & m = & 1803.042
\end{array}$$

$$\begin{array}{rrrl}
R = & 5732.010 & & \\
M = & \underline{1807.752} & & \\
\log & 7539.762 = & 3.8773576 & \\
\text{“} & 3924.258 = & \underline{3.5937575} & \\
 & & 2)\underline{7.4711151} & \\
XO = & 5439.483 & 3.7355576 &
\end{array}$$

$$\begin{array}{rrrl}
r = & 5727.300 & & \\
M = & \underline{1807.752} & & \\
\log & 7535.052 = & 3.8770863 & \\
\text{“} & 3919.548 = & \underline{3.5932361} & \\
 & & 2)\underline{7.4703224} & \\
YO = & 5434.520 & 3.7351612 &
\end{array}$$

$$\begin{array}{rrr|rrr}
R = & 5732.010 & & r = & 5727.300 & \\
m = & \underline{1803.042} & & m = & \underline{1803.042} & \\
\log & 7535.052 = & 3.8770863 & \log = & 7530.342 = & 3.8768147\\
 & 3928.968 = & \underline{3.5942786} & \log = & 3924.258 = & \underline{3.5937575}\\
 & & 2)\underline{7.4713649} & & & 2)\underline{7.4705722}\\
WE = & 5441.048 & 3.7356825 & ZE = & 5436.084 & 3.7352861
\end{array}$$

$$\begin{array}{rrr}
5439.483 & 5441.048 & 5441.048\\
\underline{5434.520} & \underline{5436.084} & \underline{5439.483}\\
4.963 & 4.964 & 1.565
\end{array}$$

$XY = 4'\ 11\frac{5}{8}''$ $WZ = 4'\ 11\frac{9}{16}''$ $WX = 4'\ 11\frac{9}{16}''$

$$\begin{array}{rrr}
5436.084 & 5441.048 & 5439.483\\
\underline{5434.520} & \underline{5434.520} & \underline{5436.084}\\
1.564 & 6.528 & 3.399
\end{array}$$

$ZY = 4'\ 11\frac{9}{16}''$ $WY = 8'\ 0\frac{9}{16}''$ $XZ = 5'\ 9\frac{11}{16}''$

Plot the steam crossing to a large scale from the original data. Plot the curved track and the tangents to the rails at each frog-point. The plot being given to the pattern-maker serves for laying out the proper bevels for the templates.

Where a street railway crosses a steam railroad track the rails for the street railway are cut to fit the railroad rails, which are not cut; therefore, allowances must be made inside the steam railroad track for width of flangeway and width of head of guard rail, with the allowance for bevel for the sum of these two widths, and outside the steam railroad track allowance is to be made for the riser or third rail with its bevel. The quickest and easiest way of finding these bevel distances is to draw lines parallel with the straight rails and distant from them the width of the flangeway, plus the width of the head of the guard rail and the width of the riser rail, these widths being taken full size. Measure the distances on the tangents drawn from each of the frog-points from the frog-point to the intersection of the tangents with these parallel lines, thus obtaining the bevel cut-offs for the street railway rails.

60. (Fig. 59) Two curved tracks crossing each other.

Figure the distance $OQ = a$ between the centers of the two curves.

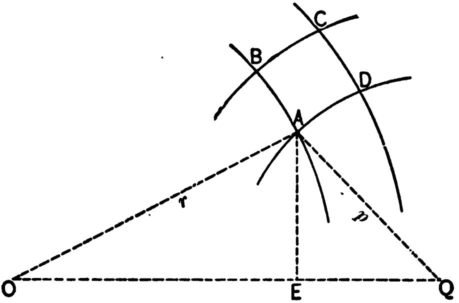

FIG. 59

In ΔOAQ, $OA = r$, $QA = p$, and $OQ = a$.

$$p^2 = r^2 + a^2 - 2\,ra\cos AOQ$$

$$r\cos AOQ = \frac{r^2 - p^2 + a^2}{2a} = \frac{(r+p)\,(r-p)}{2\,a} + \frac{a}{2}$$

but $r\cos AOQ = OE$; calling this the abscissa, x, for A we have

$$x' = \frac{(r+p)\,(r-p)}{2\,a} + \frac{a}{2};$$

$$y = AE = \sqrt{r^2 - x^2} = \sqrt{(r+x)\,(r-x)}$$

By analogy, the formulæ for co-ordinates of B, C, and D:

$$B\ldots X'' = \frac{(r+P)\,(r-P)}{2\,a} + \frac{a}{2};\ y'' = \sqrt{(r+x'')\,(r-x'')}$$

$$C\ldots X''' = \frac{(R+P)\,(r-P)}{2\,a} + \frac{a}{2};\ y''' = \sqrt{(R+x''')\,(R-x''')}$$

$$D\ldots Xiv = \frac{(R+p)\,(R-p)}{2\,a} + \frac{a}{2};\ yiv = \sqrt{(R+xiv)\,(R-xiv)}$$

Using the table of squares, the lengths BC, AB, AC, etc., may be obtained from

$$AB = \sqrt{(x' - x'')^2 + (y'' - y')^2}, \text{ etc.}$$

If there is any appreciable difference in length between the arc and chord it may be found from (4).

In a similar manner to problem 59 plot the crossing with tangents to both curves at each of the frog-points. Draw lines parallel with the tangents to the steam railroad track intersecting the tangents to the street railway track and measure the cut-offs as before.

If the frog-angles are required:

$$\sin OAE = \frac{x}{r};\ \cos OAE = \frac{y}{r}$$

$$\sin QAE = \frac{a - x}{p};\ \cos QAE = \frac{y}{p}$$

$$\sin A = \sin (OAE + QAE) = \sin OAE \cos QAE + \sin QAE \cos OAE$$

$$\sin A = \frac{xy + y(a - x)}{rp} = \frac{ay}{rp}$$

By analogy,

$$\sin B = \frac{ay^{ii}}{rP}$$

$$\sin C = \frac{ay^{iii}}{RP}$$

$$\sin D = \frac{ay^{iv}}{Rp}$$

Example: $a = 3788.60$ $R_o = 3819.835$, $P_o = 1146.285$

$$\begin{array}{rrcrr} & 3819.835 & \qquad & & 1146.285 \\ & \underline{2.355} & & & \underline{2.355} \\ R = & 3822.190 & & P = & 1148.640 \\ r = & 3817.480 & & p = & 1143.930 \end{array}$$

$$\begin{array}{rl} \log 3788.60 = & 3.5784788 \\ 2 = & \underline{0.3010300} \\ & 3.8795088 \\ \text{colog } 2a = & 6.1204912 \end{array}$$

$$\begin{array}{rrr} r = & 3817.48 & \\ p = & \underline{1143.93} & \\ \log & 4961.41 = & 3.6956051 \\ \text{"} & 2673.55 = & 3.4270883 \\ \text{colog } 2a = & & \underline{6.1204912} \\ & 1750.590 & 3.2431846 \\ \tfrac{1}{2}a = & \underline{1894.300} & \\ xA = & 3644.890 & \end{array}$$

$$\begin{array}{rr} r = & 3817.48 \\ x = & \underline{3644.89} \\ \log 7462.37 = & 3.8728768 \\ \text{"}\ 172.59 = & \underline{2.2370156} \\ & 2)\underline{6.1098924} \\ & 3.0549462 \\ yA = & 1134.870 \end{array}$$

$$\begin{aligned}
r &= 3817.48\\
P &= \underline{1148.64}\\
\log\ 4966.12 &= 3.6960172\\
\log\ 2668.84 &= 3.4263225\\
\text{colog}\ 2\,a\ &= \underline{6.1204912}\\
1749.166\quad & \ 3.2428309\\
\tfrac{1}{2}\,a &= \underline{1894.300}\\
x^B &= 3643.466
\end{aligned}
\qquad
\begin{aligned}
r &= 3817.480\\
x &= \underline{3643.466}\\
\log\ 7460.946 &= 3.8727938\\
\log\ 174.014 &= \underline{2.2405842}\\
&\ \ 2\overline{)6.1133780}\\
&\ \ \ \ 3.0566890\\
Y^B &= 1139.434
\end{aligned}$$

$$\begin{aligned}
R &= 3822.19\\
p &= \underline{1143.93}\\
\log\ 4966.12 &= 3.6960172\\
\text{“}\ \ 2678.26 &= 3.4278527\\
\text{colog}\ 2\,a\ &= \underline{6.1204912}\\
\tfrac{1}{2}\,a = 1755.339 & \ 3.2443611\\
\underline{1894.300}\\
x^D &= 3649.639
\end{aligned}
\qquad
\begin{aligned}
R &= 3822.190\\
x &= \underline{3649.639}\\
\log\ 7471.829 &= 3.8734269\\
\log\ 172.551 &= \underline{2.2369175}\\
&\ \ 2\overline{)6.1103444}\\
&\ \ \ \ 3.0551722\\
y^D &= 1135.460
\end{aligned}$$

$$\begin{aligned}
R &= 3822.19\\
P &= \underline{1148.64}\\
\log\ 4970.83 &= 3.6964289\\
\text{“}\ \ 2673.55 &= 3.4270883\\
\text{colog}\ 2\,a\ &= \underline{6.1204912}\\
1753.915\quad & \ 3.2440084\\
\tfrac{1}{2}\,a &= \underline{1894.300}\\
x^C &= 3648.215
\end{aligned}
\qquad
\begin{aligned}
R &= 3822.190\\
x &= \underline{3648.215}\\
\log\ 7470.405 &= 3.8733442\\
\text{“}\ \ 173.975 &= \underline{2.2404869}\\
&\ \ 2\overline{)6.1138311}\\
y^C = 1140.028 &\ \ \ \ 3.0569156
\end{aligned}$$

$$\begin{array}{rrr}
3644.890 & 3649.639 & 3648.215\\
\underline{3643.466} & \underline{3643.466} & \underline{3643.466}\\
(x^A - x^B) = \ 1.424 & (x^D - x^B) = \ 6.173 & (x^C - x^B) = \ 4.749
\end{array}$$

$$\begin{array}{rrr}
1139.434 & 1139.434 & 1140.028\\
\underline{1134.870} & \underline{1136.460} & \underline{1139.434}\\
(y^B - y^A) = \ 4.564 & (y^B - y^D) = \ 3.974 & (y^C - y^B) = \ 0.594
\end{array}$$

$$\begin{array}{lll}
1.424^2 = \ \ 2.0278 & 6.173^2 = 38.1059 & 4.749^2 = 22.5530\\
4.564^2 = \underline{20.8301} & 3.974^2 = \underline{15.7927} & 0.594^2 = \ \ \underline{.3014}\\
\qquad\quad\ \ 22.8579 & \qquad\quad\ 53.8986 & \qquad\quad\ 22.8544
\end{array}$$

$$AB = 4'\ 9\tfrac{3}{8}'' \qquad BD = 7'\ 4\tfrac{1}{8}'' \qquad BC = 4'\ 9\tfrac{3}{8}''$$

$$\begin{array}{rrr}
3649.639 & 3648.215 & 3649.639\\
\underline{3644.890} & \underline{3644.890} & \underline{3648.215}\\
(x^D - x^A) = \ 4.749 & (x^C - x^A) = \ 3.325 & (x^D - x^C) \quad 1.424
\end{array}$$

$$\begin{array}{rrr}
1135.460 & 1140.028 & 1140.028\\
\underline{1134.870} & \underline{1134.870} & \underline{1135.460}\\
(y^D - y^A) = \ 0.590 & (y^C - y^A) = \ 5.158 & (y^C - y^D) = \ 4.568
\end{array}$$

$$\begin{array}{rr} 4.749^2 = & 22.5530 \\ 0.590^2 = & .3481 \\ \hline & 22.9011 \end{array} \qquad \begin{array}{rr} 3.325^2 = & 11.0556 \\ 5.158^2 = & 26.6050 \\ \hline & 37.6606 \end{array} \qquad \begin{array}{rr} 1.424^2 = & 2.0278 \\ 4.568^2 = & 20.8666 \\ \hline & 22.8944 \end{array}$$

$$AD = 4'\ 9\tfrac{7}{16}'' \qquad AC = 6'\ 1\tfrac{5}{8}'' \qquad CD = 4'\ 9\tfrac{7}{16}''$$

$$\begin{array}{rl} \log a = & 3.5784788 \\ \log y = & 3.0549462 \\ \text{colog } r = & 6.4182233 \\ \text{colog } p = & 6.9416005 \\ \hline A = 79^\circ\ 55'\ 25'' = & 9.9932488 \end{array}$$

The old method of solving this problem is to figure the angles from three sides given for each frog-point, to subtract successively the angles and figure the arcs from these subtractions, and to figure the diagonals from two sides and included angle. In most cases the radii are so long as to necessitate looking up angles to tenths or hundredths of a second; a much longer process.

CROSS-OVERS FOR STREET RAILWAYS

61. (Fig. 60) Cross-over on straight track. Given the common radius, R_o, the distance between track centers, a, and the length of cross-over, $AB = b$, to find the angle O and the distance DE.

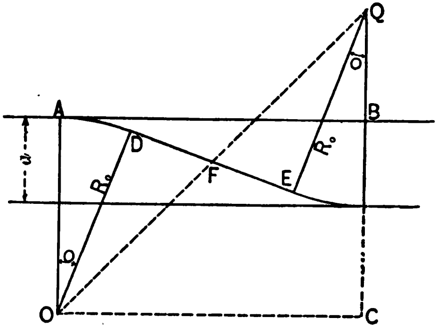

FIG. 60

$$OC = b, \text{ and } QC = 2\ R_o - a$$

$$\tan OQC = \frac{OC}{QC} = \frac{b}{2\ Ro - a}$$

$$OF = \tfrac{1}{2}\ OQ = \tfrac{1}{2}\ QC \sec OQC$$

$$\cos EQF = \frac{R_o}{QF}$$

$$DE = 2\ \overline{QF} \sin EQF$$

$$O = OQC - EQF$$

62. (Fig. 61) Cross-over on plain curve. Given the radius of the main curve, R_o, the distance between track centers, a, the radius of the switching curves, P_o, and the angle, Q, of one of the switching curves, to find the other switching curve angle, T, the distance BC, and the angle O.

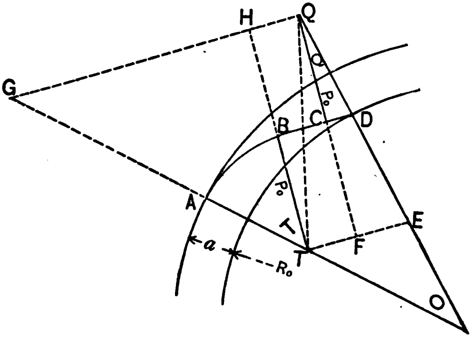

FIG. 61

Draw TE parallel with BC.

$$
\begin{aligned}
OE &= OQ - QE = R_o + P_o - 2\,P_o \sec Q \\
OT &= R_o + a - P_o \\
OET &= 90^\circ + Q \\
\sin OTE &= \frac{\cos Q}{OT}\,OE \\
T &= 90^\circ - OTE \\
O &= T - Q \\
TE &= \frac{OT}{\cos Q}\sin O \\
BC &= TF = TE - 2\,P_o \tan Q
\end{aligned}
$$

If T is given in place of Q, draw GQ parallel with BC.

$$
\begin{aligned}
OG &= OA - AT + TG = R_o + a - P_o + 2\,P_o \sec T \\
OQ &= R_o + P_o \\
OGQ &= 90^\circ - T \\
\sin OQG &= \frac{\cos T}{OQ}\,OG \\
Q &= OQG - 90^\circ \\
O &= T - Q \\
GQ &= \frac{OQ}{\cos T}\sin O \\
BC &= HQ = GQ - 2\,P_o \tan T
\end{aligned}
$$

If O is given in place of Q or T,

$$
\begin{aligned}
OQ &= R_o + P_o \\
OT &= R_o + a - P_o
\end{aligned}
$$

In (Fig. 61) $\Delta\ OQT$, having OQ, OT, and O, figure QT and TQO.

$$\cos TQF = \frac{2\,P_o}{QT}$$

$$BC = TF = 2\,P_o \tan TQF$$

$$Q = TQO - TQF$$

$$T = O + Q$$

63. (Fig. 62) Cross-over from the inside track of a curve to the tangent.

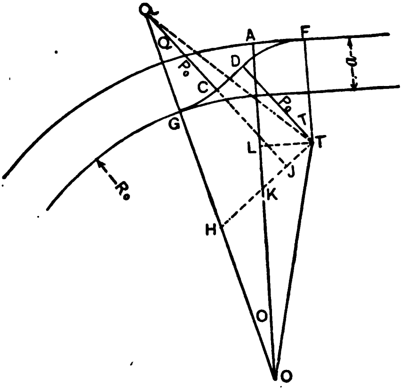

FIG. 62

Given the radius of the main curve, R_o, the radius of the switching curves, P_o, the distance between track centers, a, the angle, Q, of one of the switching curves, and the distance, AF, on the tangent, from the point of switch to the P. C. of the main curve, to find the other switching curve angle, T, the distance CD, and the angle O.

Draw HT parallel with CD.

$$\tan AOT = \frac{AF}{R_o + a - P_o}$$

$$OT = (R_o + a - P_o) \sec AOT$$

$$OH = OQ - QH = R_o + P_o - 2\,P_o \sec Q$$

$$OHT = 90^\circ + Q$$

$$\sin HTO = \frac{\cos Q}{OT}\,OH$$

$$T = 90^\circ - HTO - AOT$$

$$O = T - Q$$

$$HT = \frac{OT}{\cos Q} \sin TOH$$

$$CD = JT = HT - 2\,P_o \tan Q$$

If O is given in place of AF,

$$
\begin{aligned}
T &= O + Q \\
OH &= OQ - QH = R_o + P_o - 2\ P_o \sec Q \\
OHT &= 90^\circ + Q \\
HKO &= 90^\circ - T \\
HK &= \frac{OH}{\cos T} \sin O \\
OK &= \frac{OH}{\cos T} \cos Q \\
KL &= OA - AL - OK = R_o + a - P_o - OK \\
AF &= LT = KL \cot T
\end{aligned}
$$

$$CD = JT = HT - HJ = HK + KL \operatorname{cosec} T - 2\ P_o \tan Q$$

If O and AF are given in place of Q and T,

$$\tan AOT = \frac{AF}{Ro + a - P_o}$$

$$OT = (R_o + a - P_o) \sec AOT$$

In Δ OQT, having $OQ = R_o + P_o$, OT, and $QOT = O + AOT$, figure QT and OQT.

$$\cos JQT = \frac{2\ P_o}{QT}$$

$$
\begin{aligned}
CD &= JT = 2\ P_o \tan JQT \\
Q &= OQT - JQT \\
T &= O + Q
\end{aligned}
$$

64. (Fig. 63) Cross-over from the outside track of a curve to the tangent.

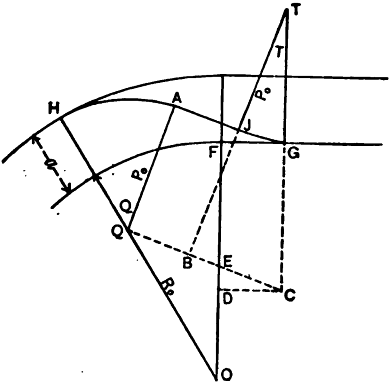

FIG. 63

Given the radius of the main curve, R_o, the radius of the switching curves, P_o, the distance between track centers, a, the angle, T, of one of the switching curves and the distance, FG, on the tangent from the point of switch to the P. C. of the main curve, to find the other switching curve angle, Q, the distance AJ, and the angle O.

Draw QC parallel with AJ, and DC perpendicular to OF.

$$DC = FG$$
$$ECD = T$$
$$DE = DC \tan T$$
$$EC = DC \sec T$$
$$TC = 2\,P_o \sec T$$
$$BC = 2\,P_o \tan T$$
$$OE = R_o + P_o - TC + DE$$
$$OQ = R_o + a - P_o$$
$$QEO = 90^\circ + T$$
$$\sin OQE = \frac{\cos T}{OQ}\,OE$$
$$Q = 90^\circ - OQE$$
$$O = Q - T$$
$$QE = \frac{OQ}{\cos T}\sin O$$
$$AJ = QB = QE + EC - BC$$

If O is given in place of FG,

$$Q = O + T$$
$$OQ = R_o + a - P_o$$

$$OQE = 90^\circ - Q$$
$$QEO = 90^\circ + T$$

$$QE = \frac{OQ}{\cos T} \sin O$$

$$OE = \frac{OQ}{\cos T} \cos Q$$

$$TC = 2\, P_o \sec T$$
$$BC = 2\, P_o \tan T$$
$$DE = TC + OE - R_o - P_o$$
$$FG = DC = DE \cot Q$$
$$EC = DE \operatorname{cosec} T$$
$$AJ = QB = QE + EC - BC$$

65. (Fig. 64) Cross-over on a compound curve.

Given the radii of the compound curves, $OL = S_o$ and $TE = R_o$, the radii of the switching curves, P_o, the distance between track centers, a, the switching curve angle, Q, and the angle, T, to find the other switching curve angle, H, the distance BC, and the angle O.

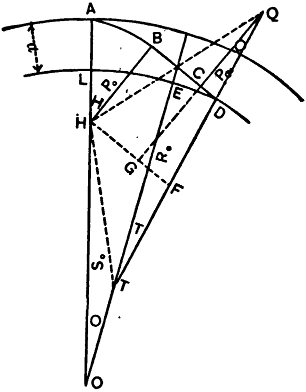

FIG. 64

Draw HF parallel with BC.

$$QF = 2\ P_o \sec Q$$
$$GF = 2\ P_o \tan Q$$
$$TF = Ro + P_o - QF$$

In Δ OTF, having T, TF, and$_o$ $OT = So - R_o$, figure OF, and TFO.

$$HFO = 90° + Q + TFO$$

In Δ OHF, having OF, HFO, and $OH = S_o + a - P_o$, figure HF, HOF, and FHO.

$$BC = HG = HF - GF$$
$$H = 90° - FHO$$
$$O = H - Q - T$$

If O is given in place of T,

$$QF = 2\ P_o \sec Q$$
$$GF = 2\ P_o \tan Q$$
$$TF = R_o + Po - QF$$

In Δ HTO, having O, $HO = S_o + a - P_o$ and $OT = S_o - R_o$, figure HT and OHT.

$$HFT = 90° + Q$$

In Δ HTF, having HFT, TF, and HT, figure HF, FHT, and HTF.

$$BC = HG = HF - GF$$

$$H = 90^\circ - FHT - OHT$$
$$T = H - O - Q$$

If O and T are given in place of Q and H.

In $\Delta\, OHT$, having O, $OH = S_o + a - P_o$ and $OT = S_o - R_o$, figure HT, and HTO.

$$HTQ = 180^\circ + T - HTO$$

In $\Delta\, HTQ$, having HTQ, HT, and $TQ = R_o + P_o$, figure QH, and HQF.

$$\cos HQG = \frac{2\, P_o}{HQ}$$

$$BC = HG = 2\, P_o \tan HQG$$
$$Q = HQT - HQG$$
$$H = T + O + Q$$

SPLIT-SWITCH SPECIAL WORK

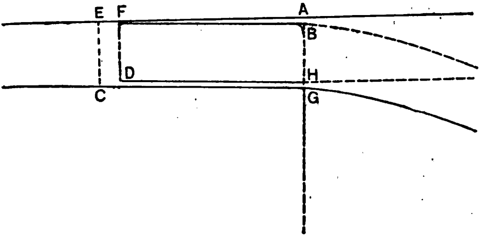

FIG. 65

The throw of a split switch is the distance, AB, at its heel between the gage lines of the main rail and switch rail. The throw varies with different makers from five to six inches; in the table the throw is taken as five and one-quarter inches. The toe of a switch-point is not brought to a true point, but is left blunt; in the table this is taken as one-quarter inch. Split-switch points are usually fifteen feet long, and are so considered in the table. Owing to the bluntness of the toe the theoretical switch-point, c, is ahead of the toe, varying with the length of the switch-point, its throw, and the bluntness of the toe. It is at the theoretical switch-point, C, in the figure, that the main rail is bent at an angle, making CG parallel with EB.

The switch-angle is the angle whose sine is the throw divided by the theoretical switch length. As split-switch points are made from tee-rail planed to fit the main rail, and as sufficient amount of the point must be in contact with the rail when being ridden over, they must be curved to fit a curved track; thus the switch-angle and throw are the same for similar switch-points whether on straight or curved track.

The curve of the branch-off usually commences at the heel of the switch-points, and as the radial line at this point is perpendicular to the switch-points, it makes an angle of ninety degrees minus the switch-angle with the main track, thus throwing the toe at *F* further away from the theoretical switch-point *E* than *C* is from *D* by *gX* sin *S*; *g* being the gage and *S* the switch-angle. In the table, the switch throw being $5\frac{1}{4}''$, the length of the point being 15′ 0″, and the bluntness of point being $\frac{1}{4}''$ the theoretical switch-point is ahead of the actual switch-point by $15 \times 12 \times \frac{1}{4} \div 5 = 9''$, considered at the center between tracks. The switch-angle is the angle whose sine is $5\frac{1}{4}'' \div (15 \times 12 + 9) = 1^\circ\ 36'$ approximately. This angle, 1° 36′, is taken as the switch-angle in the table. The gage being taken as 4.71 and the switch-angle as 1° 36′, make a difference between *EF* and *CD* of $1\frac{5}{8}''$, which makes $EF = 9\frac{13}{16}''$, and $CD = 8\frac{3}{16}''$.

Standard frogs are assigned numbers corresponding to the number of times the spread is contained in the distance from the frog-point to the middle of the spread.

To find the angle of a frog whose number is given, take twice the angle whose cotangent is twice the frog-number. The length of frog-arm is determined by the amount of clearance necessary between the angle-bar fish-plates for bolting the frog to the main rail.

For A.S.C.E. Section 75 lb. tee rail, and for smaller sections:

Arm Length	Limit for Outer Arm	Limit for Inner Arm
2′ 6″	31° 0′	16° 0′
3′ 0″	24° 0′	12° 0′
3′ 6″	19° 0′	9° 40′
4′ 0″	16° 0′	8° 0′
4′ 6″	13° 40′	7° 0′
5′ 0″	12° 0′	6° 0′
6′ 0″	9° 30′	4° 50′
7′ 0″	8° 0′	
9′ 0″	6° 0′	
10′ 0″	5° 20′	

In split-switch branch-offs, as the smaller the frog-angle becomes the greater the radius of the branch-off becomes; and, as the smaller the frog-angle becomes, the greater the length of frog-arm becomes, the ratio of the length of frog-arm to the radius of the branch-off curve is almost a constant quantity, and the ordinate from the frog-point to a line stretched from the ends of the frog-arms on a frog whose arms have been bent to conform with the curve of the branch-off is nearly one-quarter inch. If a branch-off is figured to run through the frog-point and a straight-armed frog is used, the frog-point will be too near the opposite rail by one-quarter inch, in just the position to catch the hammering from the passing train. For this reason, and from the fact that the curve beyond the frog, when such a curve occurs, is rarely of the same radius as the branch-off curve, the modern practice is to stop the branch-off curve at the end of the frog-arm as shown in the following figure:

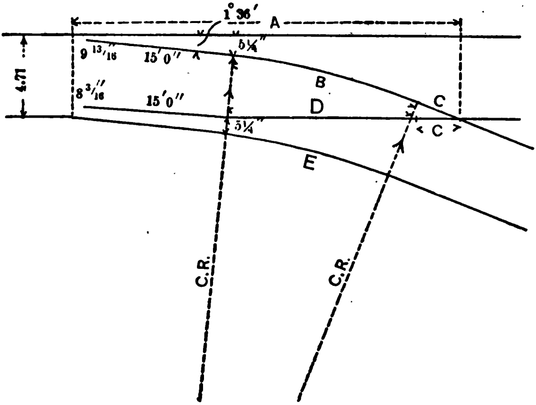

FIG. 66

F. No.	Angle	C. R.	Degree Curve	A	B			C	D			E		
					Ft.	In.	16's		Ft.	In.	16's	Ft.	In.	16's
4	14° 15′	114.01	52° 02′	44.12	25′	8″	5/16	3′ 0″	25	5	4	24	7	13
4½	12 41	148.15	39 27	47.59	29	1	6	3 0	28	10	14	28	2	7
5	11 25	182.21	31 51	50.63	31	7	8	3 6	31	5	6	30	9	13
5½	10 23	225.47	25 38	53.95	34	11	2	3 6	34	9	4	34	2	7
6	9 32	272.82	21 07	57.16	38	1	3	3 6	37	11	12	37	5	6
6½	8 48	319.91	17 59	60.08	40	6	0	4 0	40	4	12	39	10	14
7	8 10	377.60	15 13	63.14	43	6	9	4 0	43	5	8	43	0	1
7½	7 38	431.44	13 19	65.79	45	8	3	4 6	45	7	4	45	2	3
8	7 09	500.32	11 30	68.81	48	8	5	4 6	48	7	9	48	2	13
8½	6 44	573.13	10 00	71.69	51	6	12	4 6	51	6	2	51	1	10
9	6 22	650.70	8 49	74.47	54	4	1	4 6	54	3	9	53	11	4
9½	6 02	725.33	7 54	76.96	56	3	11	5 0	56	3	5	55	11	5
10	5 43	810.35	6 04½	79.55	58	4	11	5 6	58	4	7	58	0	10

The measurements given in *B*, *D*, and *E* are in feet, inches, and sixteenths.

66. (Fig. 67) Given the elements of a split switch, the gage and the radius, to find the angle of a special frog and the leads *HF* and *BF*.

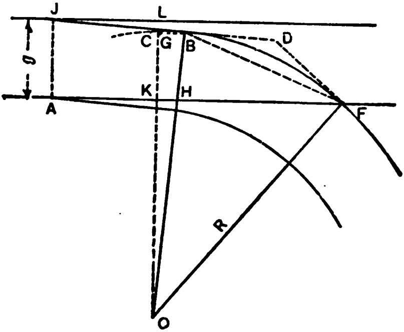

FIG. 67

Let S = switch-angle, t = throw, $l = JB$, the theoretical length of outer switch-point, and $l' = AH$, the theoretical length of inner switch-point.

$$\begin{aligned} GB &= R \sin S \\ LC &= t - R \operatorname{vers} S \\ OK &= R + LC - g \\ \cos F &= \frac{OK}{R} \\ BOF &= F - S \\ FK &= R \sin F \\ AF &= FK - GB + l \\ HF &= AF - l' \end{aligned}$$

If the chord, BF, is given in place of R,

$$\sin BFH = \frac{g - t}{BF}$$

$$\begin{aligned} BFH &= DFH - DFB = F - \tfrac{1}{2}(F - S) = \tfrac{1}{2}(F + S) \\ F &= 2\,BFH - S \\ R &= \tfrac{1}{2}\,BF \operatorname{cosec} \tfrac{1}{2}(F - S) \end{aligned}$$

This problem is convenient where full length rails are to be used for leads.

67. (Fig. 67) Given the frog-angle, switch, etc., to find the radius and leads.

By the last problem, $BFH = \frac{1}{2}(F + S)$.

$$BF = \frac{g - t}{\sin \frac{1}{2}(F + S)}$$

$$R = \frac{BF}{2 \sin \frac{1}{2}(F - S)}$$

$$= \frac{g - t}{2 \sin \frac{1}{2} (F + S) \sin \frac{1}{2} (F - S)} \text{ or}$$

$$= \frac{g - t}{\cos S - \cos F}$$

$$AF = R \ (\sin F - \sin S) + l$$

$$HF = AF - l'$$

This is the old method of running the branch-off curve through the frog-point.

68. (Fig. 68) Given the frog-angle and length of frog-arm, *BF*, the switch, etc., to find the radius and leads.

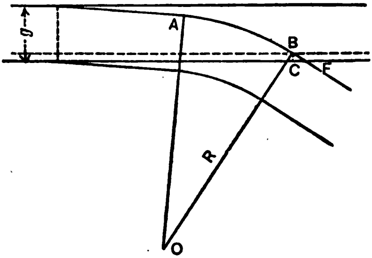

FIG. 68

$$a = BC = BF \sin F$$
$$b = CF = BF \cos F$$

By the last problem,

$$R = \frac{g - t - a}{\cos S - \cos F}$$

$$HF = R \ (\sin F - \sin S) + b + l - l'$$

69. (Fig. 69) Given the radius of a curved track, the frog, switch, etc., to find the branch-off radius and leads. (P_o = radius of branch-off curve.)

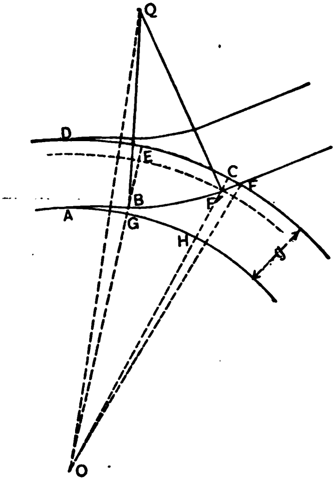

FIG. 69

Let R_o = radius of the track, F the frog-angle, and FF' the length of frog-arm.

$$a = CF' = FF' \sin F - \frac{b^2}{2R} \text{ (slide rule)}$$

$$b = CF = FF' \cos F$$

$$\sin FOC = \frac{b}{R}$$

$$F' = F - FOC$$

$$m = OB = r + t$$

$$k = OF' = R - a$$

$$OBQ = 180^\circ - S$$

$$OF'Q = 180^\circ - F'$$

In $\Delta\ OBQ,\ OQ^2 = P^2 + m^2 - 2\ Pm \cos OBQ.$
In $\Delta\ OF'Q,\ OQ^2 = P^2 + k^2 - 2\ Pk \cos OF'Q.$

Equating the values of OQ^2 and remembering that $\cos (180^\circ - A) = - \cos A$,

$$P = \frac{k^2 - m^2}{2\ (m \cos S - k \cos F')} = \frac{(k + m)\ (k - m)}{2\ (m \cos S - k \cos F')}$$

Substituting the values of m and k,

$$P = \frac{\left(R_o - \frac{a-t}{2}\right)(g - a + t)}{\left(R_o - \frac{g}{2} + t\right)\cos S - \left(R_o + \frac{g}{2} - a\right)\cos F'}$$

$$\tan OQB = \frac{m \sin S}{P + m \cos S}$$

$$\tan OQF' = \frac{k \sin F'}{P + k \cos F'}$$

$$BQF' = OQF' - OQB$$

$$BOF = F - S - BQF'$$

$$\text{arc } DE = l' + \frac{l'\, g}{r} \text{ (slide rule)}$$

$$\text{arc } DF = \text{arc } DE + R \text{ arc } BOF' + b$$

When a minus value is obtained for P in the equation, the centers of both curves are on the same side of the main track and in place of P the value of p is obtained.

70. (Fig. 70) When the frog is in the inside rail of the main track.

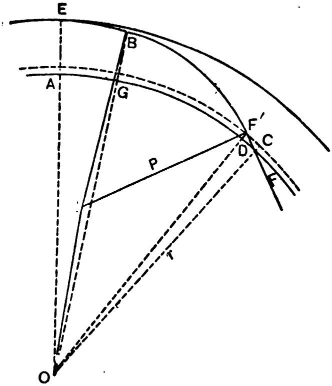

FIG. 70

$$b = DF = FF' \cos F \text{ and } a = DF' = FF' \sin F + \frac{b^2}{2R}$$

$$\sin FOF' = \frac{b}{r}$$

$$F' = F + FOF'$$

$$m = OB = R - t$$

$$k = OF' = r + a$$

$$OBQ = S$$

$$OF'Q = F'$$

In $\Delta\ OBQ$, $OQ^2 = P^2 + m^2 - 2\,Pm \cos S$.
In $\Delta\ OF'Q$, $OQ^2 = P^2 + k^2 - 2\,Pk \cos F'$.

$$P = \frac{m^2 - k^2}{2\,(m \cos S - k \cos F')} = \frac{(m + k)\,(m - k)}{2\,(m \cos S - k \cos F')} \text{ or}$$

$$= \frac{\left(R_o + \frac{a - t}{2}\right)\,(g - a - t)}{\left(R_o + \frac{g}{2} - t\right)\,\cos S - \left(R_o - \frac{g}{2} + a\right)\cos F'}$$

$$\tan QOB = \frac{P \sin S}{m - P \cos S}$$

$$\tan QOF' = \frac{P \sin F'}{k - P \cos F'}$$

$$BOF' = QOF' - QOB$$

$$BQF' = F' - S + BOF'$$

$$\text{arc } AG = l - \frac{l\,g}{R}$$

$$\text{arc } AF = \text{arc } AG + \text{arc } BOF' + b$$

71. (Fig. 71) Given the radius and arc of a curve, the switch, frog, etc., to run a branch-off with the frog on the tangent, the critical switch-point being on the outer rail of the curve.

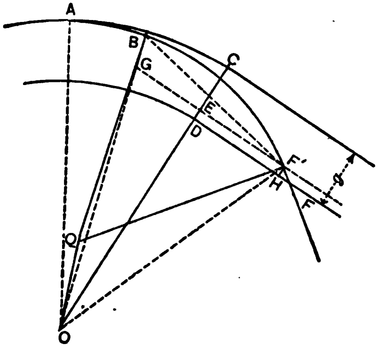

FIG. 71

Let $COA = O$ be the angle of the given arc, *Ro* the radius of the curve, *Po* the radius of the branch-off, *F* the frog, and *DF* the required distance on the tangent to fix the frog-point.

$$\begin{aligned}
b &= FH = FF' \cos F \\
a &= F'H = FF' \sin F \\
BQF' &= O + F - S \\
\text{arc } AOB &= \frac{l}{R} \\
BOE &= O - AOB \\
EG &= (r - a) \tan BOE \\
BG &= R - t - (r + a) \sec BOE \\
BF'G &= F - \tfrac{1}{2}\, BQF' = \tfrac{1}{2}\,(F + S - O) \\
BGF' &= 90^\circ + BOE \\
BF' &= \frac{BG}{\sin BF'G} \sin BGF' \\
GF' &= \frac{BG}{\sin BF'G} \sin GBF' \\
DF &= GF' - EG + b \\
P &= \tfrac{1}{2}\, BF' \operatorname{cosec} \tfrac{1}{2}\,(O + F - S)
\end{aligned}$$

If the distance, *DF*, is given in place of the angle *O*,

$$\begin{aligned}
m &= OB = R - t \\
\tan EOF' &= \frac{DF - b}{r + a} \\
k &= OF' = (r + a) \text{ see } EOF' \\
OBQ &= S \\
QF'O &= F - EOF'
\end{aligned}$$

In $\Delta\ OBQ,\ OQ^2 = P^2 + m^2 - 2\ Pm \cos S$.
In $\Delta\ OQF',\ OQ^2 = P^2 + k^2 - 2\ Pk \cos QF'O$.

$$P = \frac{(m + k)\ (m - k)}{2\ (m \cos S - k \cos QF'O)}$$

$$\tan QOB = \frac{P \sin S}{m - P \cos S}$$

$$\tan QOF' = \frac{P \sin QF'O}{k - P \cos QF'O}$$

$$BOF' = QOF' - QOB$$

$$\text{arc } AOB = \frac{l}{R}$$

$$O = AOC = AOB + BOF' - EOF'$$

$$BQF' = O + F - S$$

72. (Fig. 72) Given the radius of a curve and the distance from the point of switch on the tangent to the P. C. of the curve, to run a branch-off with the frog on the inside rail of curve.

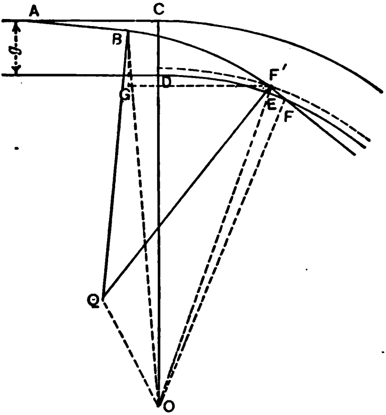

FIG. 72

$$b = EF = FF' \cos F$$

$$a = EF' = FF' \sin F + \frac{b^2}{2\ r}$$

$$\sin EOF = \frac{b}{r}$$

$$F' = QF'O = F - EOF$$

$$\tan BOC = \frac{AC - l}{R - t}$$

$$OBQ = S + BOC$$

$$m = OB = (R - t) \sec BOC$$
$$k = OF' = r + a$$

In $\Delta\, QBO$, $OQ^2 = p^2 + m^2 - 2\, Pm \cos OBQ$.
In $\Delta\, QF'O$, $OQ^2 = P^2 + k^2 - 2\, Pk \cos F'$.

$$P = \frac{(m + k)\ (m - k)}{2\,(m \cos OBQ - k \cos F')}$$

$$\tan QOB = \frac{P \sin QBO}{m - P \cos QBO}$$

$$\tan QOF' = \frac{P \sin F'}{k - P \cos F'}$$

$$COF = QOF' - QOB - BOC + EOF$$
$$BQF' = F' - S + COF$$

If the angle $DOF = O$ is given in place of AC,

$$b = EF = FF' \cos F$$

$$Q = EF' = FF' \sin F + \frac{b^2}{2\,r}$$

$$\sin EOF = \frac{b}{r}$$

$$F' = QF'O = F - EOF$$
$$BQF' = F' - S + O$$
$$BG = g - t - a + (r + a) \text{ vers } (O - EOF)$$

$$P = \frac{BG}{2 \sin \frac{1}{2}\,(S + F' + O - EOF) \sin \frac{1}{2}\,(S - F' - O + EOF)}$$

$$GF' = P\,[\sin (O + F - EOF) - \sin S]$$
$$DF' = (r + a) \sin (F - EOF)$$
$$AC = GF' - DF' + l$$

73. (Fig. 73) Given the radius of a curve and its angle, switch, frog, etc., to run a branch-off with the frog on the tangent, the critical switch-point being on the inner rail of the curve.

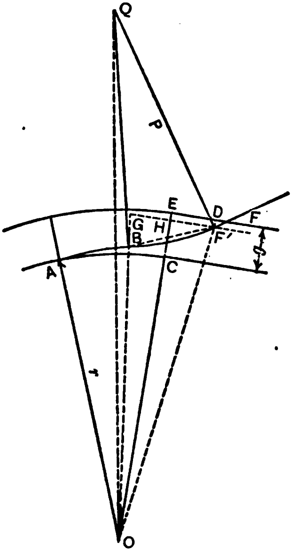

FIG. 73

Let $AOC = O$ be the given angle, R_o the radius the given curve, and P_o the radius of the branch-off.

$$b = DF = FF' \cos F$$
$$a = DF' = FF' \sin F$$
$$BQF' = F - S - O$$
$$\text{arc } AOB = \frac{1}{r}$$
$$BOE = O - AOB$$
$$GH = (R - a) \tan BOE$$
$$OG = (R - a) \sec BOE$$
$$BG = OG - r + t$$
$$BF'G = F - \tfrac{1}{2}\, BQF' = \frac{F + S - O}{2}$$
$$BGF' = 90^\circ - BOE$$
$$BF' = \frac{BG}{\sin BF'G} \sin BGF'$$
$$GF' = \frac{BG}{\sin BF'G} \sin GBF'$$

$$EF = GF' - GH + b$$
$$P = \tfrac{1}{2}\, BF' \operatorname{cosec} \tfrac{1}{2}\,(F - S - O)$$

If a minus value of BQF' is obtained, the centers of both curves are on the same side of the track and the value of p in place of P is obtained.

If the distance EF is given in place of the angle O,

$$\tan EOF' = \frac{EF - b}{R - a}$$
$$k = OF' = (R - a) \sec EOF'$$
$$m = OB = r + t$$
$$QBO = 180^\circ - S$$
$$QF'O = 180^\circ - F - EOF' = 180^\circ - F'$$

In $\Delta\, OBQ$, $OQ^2 = P^2 + m^2 + 2\, Pm \cos S$.
In $\Delta\, OF'Q$, $OQ^2 = P^2 + k^2 + 2\, Pk \cos F'$.

$$P = \frac{(k + m)\ (k - m)}{2\,(m \cos S - k \cos F')}$$
$$\tan OQB = \frac{m \sin S}{P + m \cos S}$$
$$\tan OQF' = \frac{k \sin F'}{P + k \cos F'}$$
$$BQF' = OQF' - OQB$$
$$AOC = F - BQF' - S$$

If a minus value of P is obtained, the centers of both curves are on the same side of the track and the value of p in place of P is obtained.

74. (Fig. 74) Given the radius of a curve and the distance from the point of switch on the tangent to the P. C. of the curve, to run a branch-off with the frog on the outside of the curve.

Given R_0, AC, etc.

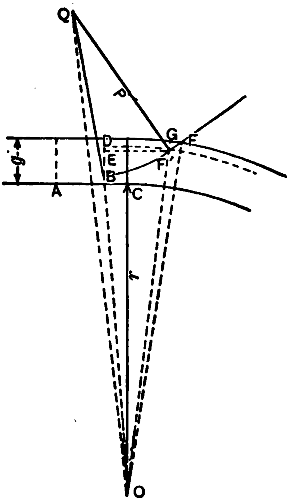

FIG. 74

$$b = GF = FF' \cos F$$

$$a = GF' = FF' \sin F - \frac{b^2}{2\,R}$$

$$\sin GOF = \frac{GF}{R}$$

$$F' = F - GOF$$

$$QF'O = 180^\circ - F'$$

$$\tan BOC = \frac{AC - l}{r + t}$$

$$m = OB = (r + t) \sec BOC$$

$$k = OF' = R - a$$

$$OBQ = 180^\circ - S + BOC = 180^\circ - B$$

In $\Delta\, OBQ$, $OQ^2 = P^2 + m^2 + 2\,Pm \cos B$.

In $\Delta\, OF'Q$, $OQ^2 = P^2 + k^2 + 2\,Pk \cos F'$.

$$P = \frac{(k + m)\;(k - m)}{2\;(m \cos B - k \cos F')}$$

$$\tan QOB = \frac{P \sin B}{m + P \cos B}$$

$$\tan QOF' = \frac{P \sin F'}{k + k \cos F'}$$

$$COF = QOF' - QOB - BOC$$

$$BQF' = F - S - COF$$

If a minus value of P is obtained, the centers of both curves lie on the same side of the track and the value of p in place of P is found.

If the angle $COF = O$ is given in place of the distance AC,

$$BQF' = F' - S - COF$$

$$BE = g - a - t - (R - a) \text{ vers } COF$$

$$BE = P\,[\cos (F' - COF) - \cos S]$$

$$P = \frac{BE}{2 \sin \frac{1}{2} (F' - O + S) \sin \frac{1}{2} (F' - O - S)}$$

$$AC = l + P \sin (F' - O) - P \sin S - (R - a) \sin O$$

$$= l + 2\,P \sin \tfrac{1}{2} (F' + O - S) \cos \tfrac{1}{2} (F' + O + S) - (R - a) \sin O$$

If a minus value of BQF' is obtained, the centers of both curves lie on the same side of the track and the value of p in place of P is found.

75. (Fig. 75) Given the radii R_o and K_o of a compound curve and the angle or arc of one curve fixing the point of switch, to run a branch-off with the frog on the inside rail of the other curve; the switch, frog, etc., being given. Let $ATC = T$ be the angle given.

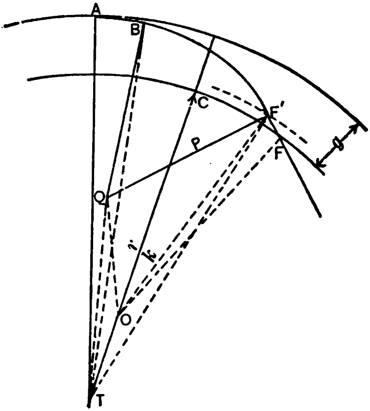

FIG. 75

$$b = FF' \cos F$$

$$a = FF' \sin F + \frac{b^2}{2\,k}$$

$$\sin FOF' = \frac{b}{r}$$

$$F' = OFQ = F - FOF'$$

$$\text{arc } ATB = \frac{l}{R}$$

$$BTC = T - ATB$$

$$n = OF' = k + a$$

In $\Delta\ TBO$, having $TB = R - t$, $TO = R_o - K_o$ and BTC, figure $m = OB$, BOC, and TBO.

$$B = QBO = S + TBO$$

In $\Delta\ OBQ$, $OQ^2 = P^2 + m^2 - 2\,Pm \cos B$.
In $\Delta\ OF'Q$, $OQ^2 = P^2 + n^2 - 2\,Pn \cos F'$.

$$P = \frac{(m + n)\ (m - n)}{2\ (m \cos B - n \cos F')}$$

$$\tan QOB = \frac{P \sin B}{m - P \cos B}$$

$$\tan QOF' = \frac{P \sin F'}{n - P \cos F'}$$

$$COF = QOF' - QOB - BOC + FOF'$$
$$BQF' = F + COF + ATC - S$$

If the angle O in place of T is given.

In the Δ TOF', having $TO = R_0 - K_0$, $OF' = K + a$, and $COF' = F - FOF'$, find $n = TF'$, $OF'T$, and OTF'.

$$QF'T = F' + OF'T$$
$$m = TB = R - t$$
$$QBT = S$$

In Δ QBT, $QT^2 = P^2 + m^2 - 2\ Pm \cos S$.
In Δ $QF'T$, $QT^2 = P^2 + n^2 - aPn \cos QF'T$.

$$P = \frac{(m + n)\ (m - n)}{2\ (m \cos S - n \cos QF'T}$$

$$\tan QTB = \frac{P \sin S}{m - P \cos S}$$

$$\tan QTF' = \frac{P \sin QF'T}{n - P \cos QF'T}$$

$$\text{arc } ATB = \frac{l}{R}$$

$$ATC = QTF' - QTB + ATB - OTF'$$
$$BQF = F - S + ATC + COF$$

76. (Fig. 76) Given the radii R_o and K_o of a compound curve and the angle, ATC, of one curve, fixing the point of switch, to run a branch-off with the frog on the outside rail of the other curve; the switch, frog, etc., being given.

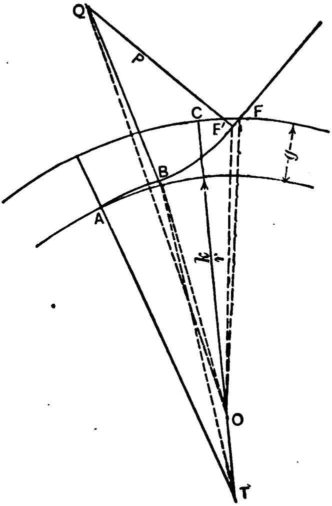

FIG. 76

$$b = FF' \cos F$$

$$a = FF' \sin F - \frac{b^2}{2\,R}$$

$$\sin FOF' = \frac{b}{R}$$

$$\text{arc } ATB = \frac{l'}{k}$$

In $\Delta\ OTB$, having $OT = K_o - R_o$, $TB = k + t$, and $BTC = ATC - ATB$, find $m = OB$, TBO, and BOC.

$$QBO = 180^\circ - S + TBO = 180^\circ - B$$
$$QF'O = 180^\circ - F + FOF' = 180^\circ - F'$$
$$n = OF' = R - a$$

In $\Delta\ OBQ$, $OQ^2 = P^2 + m^2 - 2\,Pm \cos QBO$.
In $\Delta\ OF'Q$, $OQ^2 = P + n^2 - 2\,Pn \cos QF'O$.

$$P = \frac{(m + n)\ (m - n)}{2\ (n \cos F' - m \cos B)}$$

$$\tan QOB = \frac{P \sin B}{m + P \cos B}$$

$$\tan QOF' = \frac{P \sin F'}{n + P \cos F'}$$

$$COF = QOF' - QOB - BOC + FOF'$$
$$BQF' = F - S - ATC - COF$$

If a minus value for P is obtained, the centers of all curves are on the same side of the track and the value of p in place of P is found.

If the angle $COF = O$ is given in place of T.

In the $\Delta\ OTF'$, having $OT = K\hat{o} - R\hat{o}$, $OF' = R - a$ and $COF' = F - FOF'$, find $n = TF'$, OTF', and $OF'T$.

$$m = TB = k + t$$
$$QBT = 180^\circ - S$$
$$QF'T = 180^\circ - F + OF'T = 180^\circ - F'$$

In $\Delta\ QTB$, $QT^2 = P^2 + m^2 - 2\ Pm \cos QBT$.
In $\Delta\ QTF'$, $QT^2 = P^2 + n^2 - 2\ Pn \cos QF'T$.

$$P = \frac{(n + m)\ (n - m)}{2\ (m \cos S - n \cos F')}$$

$$\tan QTB = \frac{P \sin S}{m + P \cos S}$$

$$\tan QTF' = -\frac{P \sin F'}{n + P \cos F'}$$

$$ATC = QTF' - QTB - OTF' + ATB$$
$$BQF' = F - S - ATB - COF$$

If a minus value for P is obtained, the centers of all curves are on the same side of the track and in place of P the value of p is found.

SPLIT-SWITCH CROSS-OVERS

Cross-over on a straight track.

77. (Fig. 77) Given a, g, F, to find GD.

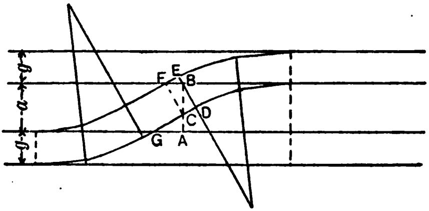

FIG. 77

$$CB = g \cos F$$
$$AC = AB - CB = a - g \cos F$$
$$GC = \frac{AC}{\sin F} = \frac{a - g \cos F}{\sin F}$$
$$GD = FE + GC$$

78. (Fig. 78) Cross-over on a plain curve.

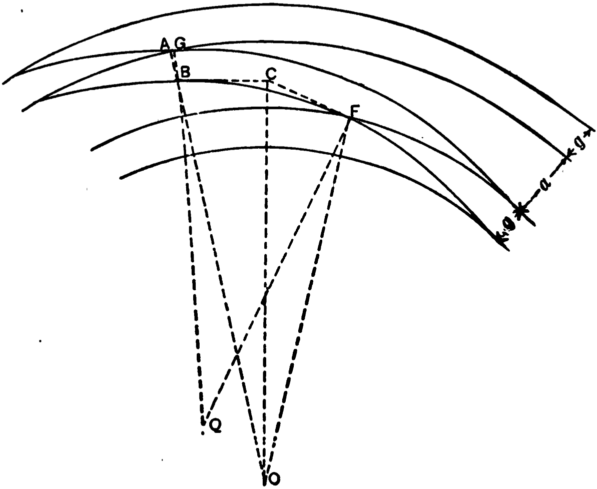

FIG. 78

Given R_0, a, g, and the frog-angles F and G.

CF is the frog-arm at F produced and BC is a line parallel with the frog-arm at G and distant from it by the gage.

Draw FQ and GQ perpendicular to the other two frog-arms. Through B, where the line GQ intersects the line BC, draw the line OA.

$$\begin{aligned} AB &= g \cos G \\ m = OB &= R + a - g \cos G \\ x = BC &= CF \\ CFO &= 90^\circ + F \\ CBO &= 90^\circ - G \end{aligned}$$

In Δ OBC, $OC^2 = m^2 + x^2 - 2\,mx \cos CBO$.
In Δ OFC, $OC^2 = R^2 + x^2 - 2\,Rx \cos CFO$.

$$x = \frac{(m + R)\ (m - R)}{2\ (R \sin F + m \sin G)}$$

$$\tan BOC = \frac{x \cos G}{m - x \sin G}$$

$$\tan COF = \frac{x \cos F}{R + x \sin F}$$

$$\begin{aligned} BOF &= BOC + COF \\ BQF &= BOF + F - G \end{aligned}$$

Having the tangents $BC = CF = x$, and the angle BQF, subtract from the tangent the length of the longer (either of F or G) outer frog-arm and figure the radius.

If $F = G$

$$x = \frac{(m + R)\ (m - R)}{2\ (m + R) \sin F} = \frac{(m - R)}{2 \sin F} = \frac{a - g \cos F}{2 \sin F}$$

$x \sin CFO = x \sin CBO$, that is, the perpendicular distance from c to OF is the same as from c to OB; therefore,

$$\tan BOC = \tan COF = \frac{x \cos F}{R + x \sin F}$$
$$BQF = BOF = 2\ BOC$$

79. (Fig. 79) Cross-over from a fixed frog on a curve to a tangent.

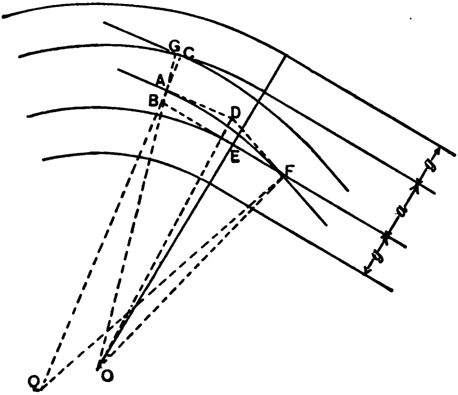

FIG. 79

Given R_o, a, g, $GOE = O$, and the frog-angles $F = C$.

Produce the frog-arm FD and draw AD parallel with the frog-arm at C and distant from it by the gage. Draw FQ and CQ perpendicular to the other two frog-arms. Through the point A, where the line QC intersects the line AD, draw OG. Produce FE to B.

$$\begin{aligned}
AG &= g \cos c \\
OA &= R + a - g \cos c \\
OB &= R \sec O \\
AB &= OA - OB \\
ADF &= 180^\circ - O \\
OBF &= 90^\circ \quad - O \\
OAF &= 90^\circ \quad - (\tfrac{1}{2}\, O + F) \\
AFB &= AFD - BFD = \tfrac{1}{2}\, O - F
\end{aligned}$$

$$AF = AB\,\frac{\sin OAF}{\sin AFB} = AB\,\frac{\cos\,(\frac{1}{2}\, O + F)}{\sin\,(\frac{1}{2}\, O - F)}$$

$$DF = AD = \frac{AF}{2 \cos \frac{1}{2}\, O}$$

$$EF = BF - BE = AF\,\frac{\cos\,(F + \frac{1}{2}\, O)}{\cos\, O} - R \tan O$$

Figure a curve tangent to AD and DF, subtracting the length of the outer frog-arm from AD or DF.

If the distance EF is given in place of the angle O,

$$\tan EOF = \frac{EF}{R}$$

$$\begin{aligned}
n &= OF = R \sec EOF \\
m &= OA = R + a - g \cos C
\end{aligned}$$

$$x = AD = DF$$
$$DFO = 90^\circ - EOF + F$$
$$DAO = 90^\circ - F$$
$$OD^2 = x^2 + m^2 - 2\,mx \cos DAO = x^2 + n^2 - 2\,nx \cos DFO$$
$$x = \frac{(m+n)(m-n)}{2\,(m \sin F - n \sin DFO)}$$
$$\tan AOD = \frac{x \cos F}{m - x \sin F}$$
$$\tan DOF = \frac{x \sin DFO}{n - x \cos DFO}$$
$$O = AOE = AOD + DOF - EOF$$
$$AQF = O$$

Figure a curve tangent to *AD* and *DF*, subtracting the length of the outer frog-arm from *AD* or *DF*.

80. (Fig. 80) Cross-over on a compound curve.
Given R_o, P_o, a, g, the frogs F and G, and the angle $GOE = O$.
Draw CF, CB, QF, QG, and TD as in the last problem.

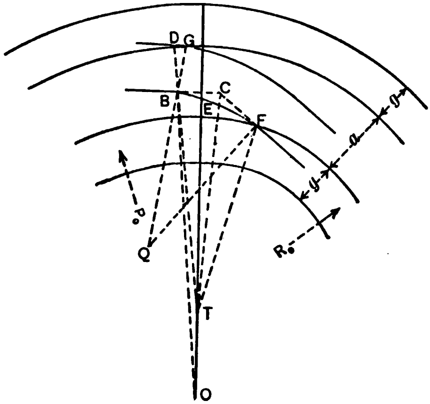

FIG. 80

$$x = BC = CF$$
$$OB = P + a - g \cos G$$
$$OT = P_o - R_o$$

In Δ OBT, having OB, OT, and O, figure $m = TB$, OBT, and BTE.

$$CBT = 90^\circ - (G + OBT)$$
$$CFT = 90^\circ + F$$

$$TC^2 = x^2 + m^2 - 2\,mx \cos CBT = x^2 + R^2 - 2\,Rx \cos CFT$$

$$X = \frac{(m + R)\,(m - R)}{2\,(m \cos CBT + R \sin F)}$$

$$\tan BTC = \frac{x \sin CBT}{m - x \cos CBT}$$

$$\tan CTF = \frac{x \cos F}{n + x \sin F}$$

$$ETF = BTC + CTF - BTE$$
$$BQF = \quad O + ETF + F - G$$

Figure a curve tangent to BC and CF, subtracting the length of the outer frog-arm from BC or CF.

WYE PROBLEMS

81. (Fig. 81) Given the three radii of a wye, the two frogs and the distance between centers of the two main tracks, to find the position of the frogs and length of arc of the third curve.

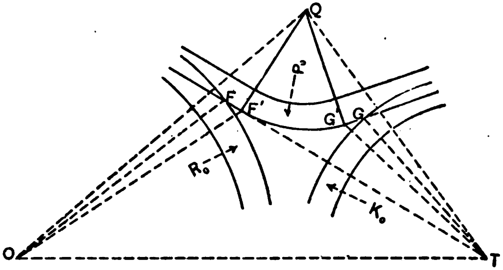

FIG. 81

Let R_o and K_o be the radii of the two main curves, and P_o the radius of the third curve.

Let F and G be the two frogs with arms FF' and GG', and OT the distance between centers of the two main curves.

$$b = FF' \cos F$$

$$a = FF' \sin F + \frac{b^2}{2R}$$

$$b' = GG' \cos G$$

$$a' = GG' \sin G + \frac{b'^2}{2K}$$

$$OF' = R + a$$

$$TG' = K + a'$$

$$\sin FOF' = \frac{b}{R}$$

$$\sin GTG' = \frac{b'}{K}$$

$$OF'Q = 180° - F - FOF'$$

$$TG'Q = 180° - G - GTG'$$

In $\Delta\ OQF'$, having OF', $QF' = P$, and $OF'Q$, figure OQ, OQF', and QOF'.

In $\Delta\ TQG'$, having TG', $QG' = P$, and $TG'Q$, figure TQ, TQG', and QTG'.

In $\Delta\ OQT$, having OQ, TQ, and OT, figure QOT, OTQ, and TQO.

$$TOF = QOT - QOF' + FOF'$$

$$OTG = OTQ - QTG' + GTG'$$

$$F'QG' = OQT - OQF' - TQG'$$

82. (Fig. 81) Given the radii of the two main curves, the distance between their centers, the two frogs, and the location of one frog, to find the radius and arc of the third curve and the location of the second frog.

Let the angle TOF fix the location of the frog F. Find, as in the last problem, OF', FOF', $TG' = n$ and GTG'.

$$TOF' = TOF - FOF'$$

In $\Delta\ TOF'$, having OF', OT, and TOF', figure $F'T = m$, $F'TO$ and $OF'T$.

$$QG'T = 180^\circ - G - GTG'$$
$$QF'T = 180^\circ - OF'T + F + FOF'$$

In $\Delta\ QF'T$, $QT^2 = P^2 + m^2 - 2\ Pm \cos QF'T$.

In $\Delta\ QG'T$, $QT^2 = P^2 + n^2 - 2\ Pn \cos QG'T$.

$$P = \frac{(n + m)\ (n - m)}{2\ (n \cos QG'T - m \cos QF'T)}$$

$$\tan F'TQ = \frac{P \sin QF'T}{m + P \cos QF'T}$$

$$\tan G'TQ = \frac{P \sin QG'T}{n + P \cos QG'T}$$

$$OTG = F'TQ - G'TQ + GTG' + FT'O$$
$$F'QG' = 180^\circ - (F + G + TOF + OTG)$$

TRANSITION CURVES

The only transition curve that can be used to advantage in special work made up in the shop is the spiral, which is composed of a series of circular arcs of equal lengths but of varying radii, the second radius being one-half of the length of the first, the third being one-third of the first radius, etc. This spiral closely approximates the cubic parabola which, as shown by Prof. C. F. Allen, in his "Railroad Curves and Earthwork," in turn approximates all other transition curves in common use. This spiral, then, may be used to replace other transition curves.

The formulæ for the co-ordinates of any P. C. C. in the spiral, letting the origin be at the P. C. and letting x' and y' represent the co-ordinates of the preceding P. C. C., R the radius of the arc under discussion, β the angle of this arc, θ the total angle, including β; are:

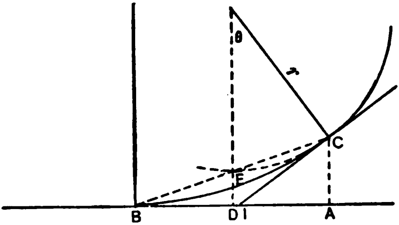

FIG. 82

$$x = x' + 2\,R \sin \tfrac{1}{2}\,(\theta - \beta) \sin \tfrac{1}{2}\,(\theta + \beta)$$
$$y = y' + 2\,R \sin \tfrac{1}{2}\,(\theta - \beta) \cos \tfrac{1}{2}\,(\theta + \beta)$$

$$x = AC$$
$$y = AB$$

Long tangent $= y - x \cot \theta = BI$
Short tangent $= x \operatorname{cosec} \theta = IC$

Deflection angle $= \tan^{-1} \dfrac{x}{y} = CBA$

Long Chord $= y \sec CBA = BC$
Offset, $a = x - r \operatorname{vers} \theta = DE$
Set-back, $b = y - r \sin \theta = BD$

The table for transition curves has been arranged for spirals, having initial arcs of 10′, 20′, and 30′; this allows for a choice of spirals, for, where the ten-minute transition gives the arcs too small the second or third may be taken. Where the initial radius is 1000 feet the first spiral gives arcs about three feet long; this is about the smallest length that should ever be used.

The value of the initial radius, R, having been obtained or assumed, the logs of x, y, long tangent, etc., may be readily obtained by adding log R to each of the logarithms given in the columns. If r is the radius of the main curve, there being no other condition imposed, and it is desired to make a transition of n arcs: $R = (n + 1)\,r$, and log a and log b may be found by adding log r to log $\dfrac{a}{r}$ and to log $\dfrac{b}{r}$ given on the line of n in the table.

If r and a are given to run a transition,

$$a = x - r \operatorname{vers} \theta$$

substituting $\dfrac{x}{R}$ for x, we have

$$R = \frac{a + r \operatorname{vers} \theta}{\dfrac{x}{R}}$$

In the column $\dfrac{a}{r}$ of the desired transition find the nearest value for log a − log r. On the same line find θ and log $\dfrac{x}{R}$. Using these values in the last equation the value of R is obtained. Example:

Given $r = 125.00$, $a = 0.43$.

$$\begin{aligned} \log\ 0.43 &= 9.6334685 \\ \log\ 125.00 &= \underline{2.0969100} \\ &\ 7.5365585 \end{aligned}$$

In the thirty-minute transition we find log $\dfrac{a}{r} = 7.530814$, and

INCHES AND SIXTEENTHS IN DECIMALS OF A FOOT

16ths	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″
0		0833	1667	2500	3333	4167	5000	5833	6667	7500	8333	9167
1	0052	0885	1719	2552	3385	4219	5052	5885	6719	7552	8385	9219
2	0104	0938	1771	2604	3438	4271	5104	5938	6771	7604	8438	9271
3	0156	0990	1823	2656	3490	4323	5156	5990	6823	7656	8490	9323
4	0208	1042	1875	2708	3542	4375	5208	6042	6875	7708	8542	9375
5	0260	1094	1927	2760	3594	4427	5260	6094	6927	7760	8594	9427
6	0313	1146	1979	2813	3646	4479	5313	6146	6979	7813	8646	9479
7	0365	1198	2031	2865	3698	4531	5365	6198	7031	7865	8698	9531
8	0417	1250	2083	2917	3750	4583	5417	6250	7083	7917	8750	9583
9	0469	1302	2135	2969	3802	4635	5469	6302	7135	7969	8802	9635
10	0521	1354	2188	3021	3854	4688	5521	6354	7188	8021	8854	9688
11	0573	1406	2240	3073	3906	4740	5573	6406	7240	8073	8906	9740
12	0625	1458	2292	3125	3958	4792	5625	6458	7292	8125	8958	9792
13	0677	1510	2344	3177	4010	4844	5677	6510	7344	8177	9010	9844
14	0729	1563	2396	3229	4063	4896	5729	6563	7396	8229	9063	9896
15	0781	1615	2448	3281	4115	4948	5781	6615	7448	8281	9115	9948
	0″	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″

MIDDLE ORDINATES FOR 10-FT. CHORDS

32ds	Radius	16ths	Radius	32ds	Radius	16ths	Radius	32ds	Radius	16ths	Radius
0″–0		1″–0	150.04	2″–0	75.08	3″–0	50.12	4″–0	37.67	5″–0	30.21
1	4807.67		145.42	1	73.93		49.64	1	37.37		30.02
2	2399.25	1	141.22	2	72.81	1	49.10	2	37.09	1	29.84
3	1600.50		137.18	3	71.73		48.61	3	36.81		29.66
4	1200.75	2	133.40	4	70.68	2	48.13	4	36.54	2	29.48
5	960.08		129.78	5	69.66		47.66	5	36.27		29.31
6	800.25	3	126.36	6	68.66	3	47.19	6	35.99	3	29.12
7	682.25		123.20	7	67.70		46.73	7	35.73		28.96
8	600.01	4	120.05	8	66.76	4	46.29	8	35.47	4	28.79
9	533.34		117.12	9	65.85		45.85	9	35.21		28.62
10	480.01	5	114.34	10	64.96	5	45.42	10	34.97	5	28.46
11	436.38		111.68	11	64.10		45.00	11	34.71		28.29
12	400.02	6	109.15	12	63.25	6	44.58	12	34.47	6	28.13
13	369.25		106.72	13	62.44		44.18	13	34.22		27.97
14	342.86	7	104.41	14	61.14	7	43.78	14	33.99	7	27.81
15	320.04		102.19	15	60.86		43.39	15	33.75		27.66
16	300.02	8	100.06	16	60.10	8	43.00	16	33.52	8	27.50
17	282.38		98.02	17	59.35		42.62	17	33.26		27.35
18	266.68	9	96.06	18	58.65	9	42.25	18	33.07	9	27.20
19	253.08		94.18	19	57.94		41.89	19	32.84		27.05
20	240.04	10	92.37	20	57.25	10	41.53	20	32.62	10	26.90
21	228.59		90.59	21	56.58		41.18	21	32.41		26.76
22	218.15	11	88.96	22	55.93	11	40.83	22	32.20	11	26.61
23	209.16		87.37	23	55.29		40.49	23	31.98		26.46
24	200.21	12	85.79	24	54.66	12	40.16	24	31.78	12	26.33
25	192.03		84.29	25	54.05		39.83	25	31.57		26.19
26	184.65	13	82.83	26	53.45	13	39.50	26	31.36	13	26.05
27	177.81		81.44	27	52.86		39.18	27	31.17		25.91
28	171.46	14	80.08	28	52.29	14	38.87	28	30.97	14	25.78
29	165.53		78.77	29	51.73		38.56	29	30.78		25.64
30	160.04	15	77.50	30	51.19	15	38.26	30	30.58	15	25.51
31	154.87		76.27	31	50.65		37.96	31	30.40		25.38

LOGARITHMS IN COMMON USE

Number	Log	Number	Log	Number	Log	Number	Log
2.166	0.3356586	44.71	1.6504047	69.71	1.8432951	94.71	1.9763958
2.355	0.3719909	45.	1.6532125	70.	1.8450980	95.	1.9777236
2.50	0.3979400	45.355	1.6566252	70.355	1.8472950	95.355	1.9793435
4.333	0.6367887	47.	1.6720979	72.	1.8573325	97.	1.9867717
4.50	0.6532125	47.355	1.6753658	72.355	1.8594685	97.355	1.9883583
4.71	0.6730209	47.645	1.6780173	72.645	1.8612057	97.645	1.9896500
5.355	0.7287595	47.71	1.6786094	72.71	1.8615941	97.71	1.9899390
7.71	0.8870544	49.645	1.6958755	74.645	1.8730007	99.71	1.9987387
9.21	0.9642596	49.71	1.6964438	74.71	1.8733787	100.	2.0000000
9.71	0.9872192	50.	1.6989700	75.	1.8750613	100.355	2.0015390
10.71	1.0297895	50.355	1.7020426	75.355	1.8771121	102.355	2.0101090
13.92	1.1436392	52.	1.7160033	77.	1.8864907	102.71	2.0116127
14.42	1.1589653	52.355	1.7189582	77.355	1.8884884	104.71	2.0199882
19.92	1.2992893	52.645	1.7213571	77.645	1.8901135	105.355	2.0226551
20.42	1.3100557	52.71	1.7218930	77.71	1.8904769	107.71	2.0322560
24.645	1.3917288	54.645	1.7375504	79.645	1.9011585	144.645	2.1603034
27.	1.4313638	54.71	1.7380667	79.71	1.9015128	147.	2.1673173
27.645	1.4416166	55.	1.7403627	80.	1.9030900	147.645	2.1692187
29.645	1.4719515	55.355	1 7431569	80.355	1.9050129	150.	2.1760913
30.	1.4771213	57.	1.7558749	82.	1.9138139	152.355	2.1828567
32.	1.5051500	57.355	1.7585713	82.355	1.9156900	154.71	2.1895184
32.355	1.5099414	57.645	1.7607616	82.645	1.9172166	155.355	2.1913253
32.645	1.5138167	57.71	1.7612511	82.71	1.9175580	157.71	2 1978592
34.645	1.5396406	59.645	1.7755740	84.645	1.9276013	194.645	2.2892432
34.71	1.5404546	59.71	1.7760471	84.71	1.9279347	197.	2.2944662
35.	1.5440680	60.	1.7781513	85.	1.9294189	197.645	2.2958858
35.355	1.5484508	60.355	1.7807133	85.355	1.9312290	200.	2.3010300
37.	1.5682017	62.	1.7923917	87.	1.9395193	202.355	2.3061139
37.355	1.5723487	62.355	1.7948713	87.355	1.9412878	204.71	2.3111391
37.645	1.5757073	62.645	1.7968864	87.645	1.9427271	205.355	2.3125053
37.71	1.5764565	62.71	1.7973368	87.71	1.9430491	207.71	2.3174574
39.645	1.5981884	64.645	1.8105349	89.645	1.9525261	Arc 90°	0.1961199
39.71	1.5988999	64.71	1.8109714	89.71	1.9528409	" 1"	4.6855749
40.	1.6020600	65.	1.8129134	90.	1.9542425	" 1°	8.2418774
40.355	1.6058974	65.355	1.8152788	90.355	1.9559522		
42.	1.6232493	67.	1.8260748	92.	1.9637878		
42.355	1.6269047	67.355	1.8283698	92.355	1.9654604		
42.645	1.6298681	67.645	1.8302357	92.645	1.9668220		
42.71	1.6305296	67.71	1.8306528	92.71	1.9671266		
44.645	1.6497728	69.645	1.8428899	94.645	1.9760977		

CHORDS

′	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	′
0	0000	0175	0349	0524	0698	0872	1047	1221	1395	1569	1743	1917	2091	2264	2437	0
2	06	80	55	29	0704	78	53	27	1401	75	49	23	96	70	43	2
4	12	86	61	35	10	84	58	33	07	81	55	28	2102	76	49	4
6	17	92	66	41	15	90	64	38	13	87	61	34	08	81	55	6
8	23	98	72	47	21	96	70	44	18	92	66	40	14	87	60	8
10	29	0204	78	53	27	0901	76	50	24	98	72	46	19	93	66	10
12	35	09	84	58	33	07	82	56	30	1604	78	52	25	99	72	12
14	41	15	90	64	39	13	87	62	36	10	84	57	31	2305	78	14
16	47	21	96	70	45	19	93	67	42	16	89	63	37	10	84	16
18	52	27	0401	76	50	25	99	73	47	21	95	69	43	16	89	18
20	58	33	07	82	56	31	1105	79	53	27	1801	75	48	22	95	20
22	64	39	13	88	62	36	11	85	59	33	07	81	54	28	2501	22
24	70	44	19	93	68	42	16	91	65	39	13	86	60	33	07	24
26	76	50	25	99	74	48	22	96	71	45	18	92	66	39	12	26
28	81	56	30	0605	79	54	28	1302	76	50	24	98	72	45	18	28
30	87	62	36	11	85	60	34	08	82	56	30	2004	77	51	24	30
32	93	68	42	17	91	65	40	14	88	62	36	10	83	57	30	32
34	99	73	48	22	97	71	45	20	94	68	42	15	89	62	36	34
36	0105	79	54	28	0803	77	51	25	1500	74	47	21	95	68	41	36
38	11	85	60	34	08	83	57	31	05	79	53	27	2200	74	47	38
40	16	91	65	40	14	89	63	37	11	85	59	33	06	80	53	40
42	22	97	71	46	20	94	69	43	17	91	65	38	12	85	59	42
44	28	0303	77	51	26	1000	75	49	23	97	71	44	18	91	64	44
46	34	08	83	57	32	06	80	55	29	1703	76	50	24	97	70	46
48	40	14	89	63	38	12	86	60	34	08	82	56	29	2403	76	48
50	45	20	94	69	43	18	92	66	40	14	88	62	35	09	82	50
52	51	26	0500	75	49	23	98	72	46	20	94	67	41	14	87	52
54	57	32	06	81	55	29	1204	78	52	26	1900	73	47	20	93	54
56	63	37	12	86	61	35	09	84	58	32	05	79	53	26	99	56
58	69	43	18	92	67	41	15	89	63	37	11	85	58	32	2605	58
60	75	49	24	98	72	47	21	95	69	43	17	91	64	37	11	60

Chords

′	15°	16°	17°	18°	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	′
0	2611	2783	2956	3129	3301	3473	3645	3816	3987	4158	4329	4499	4669	4838	5008	0
2	16	89	62	34	07	79	50	22	93	64	34	4505	75	44	13	2
4	22	95	68	40	12	84	56	28	99	70	40	10	80	50	19	4
6	28	2801	73	46	18	90	62	33	4004	75	46	16	86	55	24	6
8	34	07	79	52	24	96	68	39	10	81	52	22	92	61	30	8
10	39	12	85	57	30	3502	73	45	16	87	57	27	97	67	36	10
12	45	18	91	63	35	07	79	50	22	92	63	33	4703	72	41	12
14	51	24	96	69	41	13	85	56	27	98	69	39	08	78	47	14
16	57	30	3002	75	47	19	90	62	33	4204	74	44	14	84	53	16
18	62	35	08	80	53	25	96	68	39	09	80	50	20	89	58	18
20	68	41	14	86	58	30	3702	73	44	15	86	56	25	95	64	20
22	74	47	19	92	64	36	08	79	50	21	91	61	31	4901	70	22
24	80	53	25	98	70	42	13	85	56	26	97	67	37	06	75	24
26	85	58	31	3203	76	47	19	90	61	32	4403	73	42	12	81	26
28	91	64	37	09	81	53	25	96	67	38	08	78	48	17	86	28
30	97	70	42	15	87	59	30	3902	73	44	14	84	54	23	92	30
32	2703	76	48	21	93	65	36	08	79	49	20	90	59	29	98	32
34	09	81	54	26	98	70	42	13	84	55	25	95	65	34	5103	34
36	14	87	60	32	3404	76	48	19	90	61	31	4601	71	40	09	36
38	20	93	65	38	10	82	53	25	96	66	37	07	76	46	15	38
40	26	99	71	44	16	87	59	30	4101	72	42	12	82	51	20	40
42	32	2904	77	49	21	93	65	36	07	78	48	18	88	57	26	42
44	37	10	83	55	27	99	70	42	13	83	54	24	93	63	31	44
46	43	16	88	61	33	3605	76	47	18	89	59	29	99	68	37	46
48	49	22	94	67	39	10	82	53	24	95	65	35	4805	74	43	48
50	55	27	3100	72	44	16	88	59	30	4300	71	41	10	79	48	50
52	60	33	06	78	50	22	93	65	35	06	76	46	16	85	54	52
54	66	39	11	84	56	28	99	70	41	12	82	52	22	91	60	54
56	72	45	17	89	62	33	3805	76	47	17	88	58	27	96	65	56
58	78	50	23	95	67	39	10	82	53	23	93	63	33	5002	71	58
60	83	56	29	3301	73	45	16	87	58	29	99	69	38	08	76	60

CHORDS

′	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	′
0	5176	5345	5513	5680	5847	6014	6180	6346	6511	6676	6840	7004	7167	7330	7492	0
2	82	50	18	86	53	20	86	52	17	82	46	10	73	35	98	2
4	88	56	24	91	59	25	91	57	22	87	51	15	78	41	7503	4
6	93	62	30	97	64	31	97	63	28	93	57	20	84	46	08	6
8	99	67	35	5703	70	36	6202	68	33	98	62	26	89	52	14	8
10	5204	73	41	08	75	42	08	74	39	6704	68	31	95	57	19	10
12	10	78	46	14	81	47	14	79	44	09	73	37	7200	62	24	12
14	16	84	52	19	86	53	19	85	50	15	79	42	05	68	30	14
16	21	90	57	25	92	58	25	90	55	20	84	58	11	73	35	16
18	27	95	63	30	97	64	30	96	61	25	90	53	16	79	41	18
20	33	5401	69	36	5903	70	36	6401	66	31	95	59	22	84	46	20
22	38	06	74	42	09	75	41	07	72	36	6901	64	27	90	51	22
24	44	12	80	47	14	81	47	12	77	42	06	69	32	95	57	24
26	49	18	85	53	20	86	52	18	83	47	11	75	38	7400	62	26
28	55	23	91	58	25	92	58	23	88	53	17	80	43	06	68	28
30	61	29	97	64	31	97	63	29	94	58	22	86	49	11	73	30
32	66	34	5602	69	36	6103	69	34	99	64	28	91	54	17	78	32
34	72	40	08	75	42	08	74	40	6605	69	33	97	60	22	84	34
36	77	46	13	81	47	14	80	45	10	75	39	7102	65	27	89	36
38	83	51	19	86	53	19	85	51	16	80	44	08	70	33	95	38
40	89	57	25	92	59	25	91	56	21	86	50	13	76	38	7600	40
42	94	62	30	97	64	30	96	62	27	91	55	18	81	43	05	42
44	5300	68	36	5803	70	36	6302	67	32	97	61	24	87	49	11	44
46	06	74	41	08	75	42	07	73	38	6802	66	29	92	54	16	46
48	11	79	47	14	81	47	13	78	43	08	71	35	98	60	21	48
50	17	85	52	20	86	53	18	84	49	13	77	40	7303	65	27	50
52	22	90	58	25	92	58	24	89	54	19	82	46	08	71	32	52
54	28	96	64	31	97	64	30	95	60	24	88	51	14	76	38	54
56	34	5502	69	36	6003	69	35	6500	65	29	93	56	19	81	43	56
58	39	07	75	42	09	75	41	06	71	35	99	62	25	87	48	58
60	45	13	80	47	14	80	46	11	76	40	7004	67	30	92	54	60

CHORDS

′	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	58°	59°	′
0	7654	7815	7975	8135	8294	8452	8610	8767	8924	9080	9235	9389	9543	9696	9848	0
2	59	20	80	40	99	58	15	73	29	85	40	95	48	9701	54	2
4	64	25	86	45	8304	63	21	78	34	90	45	9400	53	06	59	4
6	70	31	91	51	10	68	26	83	40	95	50	05	59	11	64	6
8	75	36	96	56	15	73	31	88	45	9101	56	10	64	17	69	8
10	81	41	8002	61	20	79	36	94	50	06	61	15	69	22	74	10
12	86	47	07	67	26	84	42	99	55	11	66	20	74	27	79	12
14	91	52	12	72	31	89	47	8804	60	16	71	25	79	32	84	14
16	97	57	18	77	36	95	52	09	66	21	76	30	84	37	89	16
18	7702	63	23	83	41	8500	57	14	71	26	81	36	89	42	94	18
20	07	68	28	88	47	05	63	20	76	32	87	41	94	47	99	20
22	13	73	34	93	52	10	68	25	81	37	92	46	99	52	9904	22
24	18	79	39	98	57	16	73	30	86	42	97	51	9604	57	09	24
26	23	84	44	8204	63	21	78	35	92	47	9302	56	10	62	14	26
28	29	90	50	09	68	26	84	41	97	52	07	61	15	67	19	28
30	34	95	55	14	73	31	89	46	9002	57	12	66	20	72	24	30
32	40	7900	60	20	78	37	94	51	07	63	17	72	25	78	29	32
34	45	06	66	25	84	42	99	56	12	68	23	77	30	83	34	34
36	50	11	71	30	89	47	8705	61	18	73	28	82	35	88	39	36
38	56	16	76	36	94	52	10	67	23	78	33	87	40	93	45	38
40	61	22	82	41	8400	58	15	72	28	83	38	92	45	98	50	40
42	66	27	87	46	05	63	20	77	33	88	43	97	50	9803	55	42
44	72	32	92	51	10	68	26	82	38	94	48	9502	55	08	60	44
46	77	38	98	57	15	73	31	87	44	99	53	07	61	13	65	46
48	82	43	8103	62	21	79	36	93	49	9204	59	12	66	18	70	48
50	88	48	08	67	26	84	41	98	54	09	64	18	71	23	75	50
52	93	54	13	73	31	89	47	8903	59	14	69	23	76	28	80	52
54	99	59	19	78	37	94	52	08	64	19	74	28	81	33	85	54
56	7804	64	24	83	42	8600	57	14	69	25	79	33	86	38	90	56
58	09	70	29	89	47	05	62	19	75	30	84	38	91	43	95	58
60	15	75	35	94	52	10	67	24	80	35	89	43	96	48	10000	60

CHORDS

′	60°	61°	62°	63°	64°	65°	66°	67°	68°	69°	70°	71°	72°	73°	74°	
0	1.0000	1.0151	1.0301	1.0450	1.0598	1.0746	1.0893	1.1039	1.1184	1.1328	1.1472	1.1614	1.1756	1.1896	1.2036	0
2	05	56	06	55	1.0603	51	98	44	89	33	76	19	60	1.1901	41	2
4	10	61	11	60	08	56	1.0903	48	94	38	81	24	65	06	46	4
6	15	66	16	65	13	61	07	53	98	42	86	28	70	10	50	6
8	20	71	21	70	18	66	12	58	1.1203	47	91	33	75	15	55	8
10	25	76	26	75	23	71	17	63	08	52	95	38	79	20	60	10
12	30	81	31	80	28	75	22	68	13	57	1.1500	42	84	24	64	12
14	35	86	36	85	33	80	27	73	18	62	05	47	89	29	69	14
16	40	91	41	90	38	85	32	78	22	66	10	52	93	34	73	16
18	45	96	46	95	43	90	37	82	27	71	14	57	98	38	78	18
20	50	1.0201	51	1.0500	48	95	42	87	32	76	19	61	1.1803	43	83	20
22	55	06	56	04	53	1.0800	46	92	37	81	24	66	07	48	87	22
24	60	11	61	09	58	05	51	97	42	86	29	71	12	52	92	24
26	65	16	66	14	62	10	56	1.1102	46	90	33	76	17	57	97	26
28	70	21	70	19	67	15	61	07	51	95	38	80	21	62	1.2101	28
30	75	26	75	24	72	20	66	11	56	1.1400	43	85	26	66	06	30
32	80	31	80	29	77	24	71	16	61	05	48	90	31	71	11	32
34	86	36	85	34	82	29	76	21	66	09	52	94	36	76	15	34
36	91	41	90	39	87	34	80	26	71	14	57	99	40	80	20	36
38	96	46	95	44	92	39	85	31	75	19	62	1.1704	45	85	24	38
40	1.0101	51	1.0400	49	97	44	90	36	80	24	67	09	50	90	29	40
42	06	56	05	54	1.0702	49	95	40	85	29	71	13	54	94	34	42
44	11	61	10	59	07	54	1.1000	45	90	33	76	18	59	99	38	44
46	16	66	15	64	12	59	05	50	95	38	81	23	64	1.2004	43	46
48	21	71	20	69	17	63	10	55	99	43	86	27	68	08	48	48
50	26	76	25	74	21	68	14	60	1.1304	48	90	32	73	13	52	50
52	31	81	30	79	26	73	19	65	09	52	95	37	78	18	57	52
54	36	86	35	84	31	78	24	69	14	57	1.1600	42	82	22	61	54
56	41	91	40	89	36	83	29	74	19	62	05	46	87	27	66	56
58	46	96	45	93	41	88	34	79	23	67	09	51	92	32	71	58
60	51	1.0301	50	98	46	93	39	84	28	72	14	56	96	36	75	60

CHORDS

′	75°	76°	77°	78°	79°	80°	81°	82°	83°	84°	85°	86°	87°	88°	89°	′
0	1.2175	1.2313	1.2450	1.2586	1.2722	1.2856	1.2989	1.3121	1.3252	1.3383	1.3512	1.3640	1.3767	1.3893	1.4018	0
2	80	18	55	91	26	60	93	26	57	87	16	44	71	97	22	2
4	84	22	59	95	31	65	98	30	61	91	20	48	76	1.3902	26	4
6	89	27	64	1.2600	35	69	1.3002	34	65	96	25	53	80	06	31	6
8	94	32	68	04	40	74	07	39	70	1.3400	29	57	84	10	35	8
10	98	36	73	09	44	78	11	43	74	04	33	61	88	14	39	10
12	1.2203	41	78	14	48	82	15	47	79	09	38	65	92	18	43	12
14	08	45	82	18	53	87	20	52	83	13	42	70	97	22	47	14
16	12	50	87	23	57	91	24	56	87	17	46	74	1.3801	27	51	16
18	17	54	91	27	62	96	29	61	92	21	50	78	05	31	55	18
20	21	59	96	32	66	1.2900	33	65	96	26	55	82	09	35	60	20
22	26	64	1.2500	36	71	05	38	69	1.3300	30	59	87	13	39	64	22
24	31	68	05	41	75	09	42	74	05	34	63	91	18	43	68	24
26	35	73	09	45	80	14	46	78	09	39	67	95	22	47	72	26
28	40	77	14	50	84	18	51	83	13	43	72	99	26	52	76	28
30	44	82	18	54	89	22	55	87	18	47	76	1.3704	30	56	80	30
32	49	86	23	59	93	27	60	91	22	52	80	08	34	60	84	32
34	54	91	28	63	98	31	64	96	26	56	85	12	39	64	89	34
36	58	96	32	68	1.2802	36	68	1.3200	31	60	89	16	43	68	93	36
38	63	1.2400	37	72	07	40	73	04	35	65	93	21	47	72	97	38
40	67	05	41	77	11	45	77	09	39	69	97	25	51	77	1.4101	40
42	72	09	46	81	16	49	82	13	44	73	1.3602	29	55	81	05	42
44	77	14	50	86	20	54	86	18	48	77	06	33	60	85	09	44
46	81	18	55	90	25	58	90	22	52	82	10	38	64	89	13	46
48	86	23	59	95	29	62	95	26	57	86	14	42	68	93	17	48
50	90	28	64	99	33	67	99	31	61	90	19	46	72	97	22	50
52	95	32	68	1.2704	38	71	1.3104	35	65	95	23	50	76	1.4002	26	52
54	99	37	73	08	42	76	08	39	70	99	27	54	81	06	30	54
56	1.2304	41	77	13	47	80	12	44	74	1.3503	31	59	85	10	34	56
58	09	46	82	17	51	85	17	48	78	08	36	63	89	14	38	58
60	13	50	86	22	56	89	21	52	83	12	40	67	93	18	42	60

DEGREE OF CURVE — RADIUS — MID. ORD. FOR 30 FT.

°	Rad.	Mo.	°	Rad.	Mo.	°	Rad.	Mo.	°	Rad.	Mo.
0° 05	68754.94	.002	05	1403.46	.080	05	709.40	.159	05	475.05	.236
10	34377.48	.003	10	1375.40	.082	10	702.18	.160	10	471.81	.238
15	22918.33	.005	15	1348.45	.083	15	695.09	.162	15	468.61	.239
20	17188.76	.007	20	1322.53	.085	20	668.16	.163	20	465.46	.242
25	13751.02	.008	25	1297.58	.087	25	681.35	.165	25	462.35	.243
30	11459.19	.010	30	1273.57	.088	30	674.69	.167	30	459.28	.245
35	9822.02	.011	35	1250.42	.090	35	668.15	.168	35	456.25	.246
40	8594.41	.013	40	1228.11	.092	40	661.74	.170	40	453.26	.248
45	7639.49	.015	45	1206.57	.093	45	655.45	.172	45	450.31	.250
50	6875.55	.016	50	1185.78	.095	50	649.27	.173	50	447.40	.251
55	6250.51	.018	55	1165.70	.097	55	643.22	.175	55	444.52	.253
1° 00	5729.65	.020	5° 00	1146.28	.098	9° 00	637.27	.177	13° 00	441.68	.255
05	5288.92	.021	05	1127.50	.100	05	631.44	.178	05	438.88	.257
10	4911.15	.023	10	1109.33	.101	10	625.71	.180	10	436.12	.258
15	4583.75	.025	15	1191.73	.103	15	620.09	.181	15	433.39	.260
20	4297.28	.026	20	1074.68	.105	20	614.56	.183	20	430.69	.261
25	4044.51	.028	25	1058.16	.106	25	609.14	.185	25	428.03	.262
30	3819.83	.029	30	1042.14	.108	30	603.80	.186	30	425.40	.264
35	3618.80	.031	35	1026.60	.110	35	598.57	.188	35	422.80	.266
40	3437.87	.033	40	1011.51	.111	40	593.42	.190	40	420.23	.268
45	3274.17	.034	45	996.87	.113	45	588.36	.191	45	417.70	.270
50	3125.36	.036	50	982.64	.114	50	583.38	.193	50	415.19	.271
55	2989.48	.038	55	968.81	.116	55	578.49	.195	55	412.72	.273
2° 00	2864.93	.039	6° 00	955.37	.118	10° 00	573.69	.196	14° 00	410.28	.274
05	2750.35	041	05	942.29	.119	05	568.96	.197	05	407.86	.276
10	2644.58.	.043	10	929.59	.121	10	564.31	.199	10	405.47	.277
15	2546.64	.044	15	917.19	.123	15	559.73	.201	15	403.11	.279
20	2455.70	.046	20	905.13	.124	20	555.23	.203	20	400.78	.281
25	2371.04	.047	25	893.39	.126	25	550.80	.205	25	398.48	.282
30	2292.01	.049	30	881.95	.128	30	546.44	.206	30	396.20	.284
35	2218.09	.051	35	870.79	.129	35	542.18	.207	35	393.95	.285
40	2148.79	.052	40	859.92	.131	40	537.92	.209	40	391.72	.287
45	2083.68	.054	45	849.32	.132	45	533.77	.210	45	389.52	.288
50	2022.41	.056	50	838.97	.134	50	529.67	.212	50	387.34	.290
55	1964.64	.057	55	828.88	.136	55	525.64	.214	55	385.19	.292
3° 00	1910.08	.059	7° 00	819.02	.137	11° 00	521.67	.216	15° 00	383.06	.294
05	1858.47	.061	05	809.40	.139	05	517.76	.217	05	380.96	.295
10	1809.57	.062	10	800.00	.141	10	513.91	.219	10	378.88	.297
15	1763.18	.064	15	790.81	.142	15	510.12	.220	15	376.82	.298
20	1719.12	.065	20	781.84	.144	20	506.38	.222	20	374.79	.300
25	1677.20	.067	25	773.07	.146	25	502.69	.224	25	372.77	.301
30	1637.28	.069	30	764.49	.147	30	499.06	.225	30	370.78	.303
35	1599.21	.070	35	756.10	.149	35	495.48	.227	35	368.81	.305
40	1562.88	.072	40	747.89	.150	40	491.96	.229	40	366.86	.307
45	1528.16	.074	45	739.86	.152	45	488.48	.231	45	364.93	.309
50	1494.95	.075	50	732.01	.154	50	485.05	.232	50	363.02	.310
55	1463.16	.077	55	724.31	.155	55	481.67	.233	55	361.13	.312
4° 00	1432.69	.079	8° 00	716.78	.157	12° 00	478.34	.235	16° 00	359.26	.313

DEGREE OF CURVE — RADIUS, ETC. (*continued*)

°	Rad.	Mo.	°	Rad.	Mo.	°	Rad.	Mo.	°	Rad.	Mo.
16° 05	357.42	.315	05	336.64	.335	05	318.16	.353	05	301.63	.373
10	355.59	.316	10	335.01	.336	10	316.71	.355	10	300.33	.375
15	353.77	.318	15	333.41	.338	15	315.28	.356	15	299.05	.377
20	351.98	.320	20	331.82	.339	20	313.86	.358	20	297.77	.378
25	350.21	.321	25	330.24	.340	25	312.45	.360	25	296.50	.380
30	348.45	.323	30	328.68	.342	30	311.06	.362	30	295.25	.381
35	346.71	.325	35	327.14	.344	35	309.67	.363	35	294.00	.383
40	344.99	.326	40	325.60	.346	40	308.30	.365	40	292.77	.384
45	343.29	.328	45	324.09	.347	45	306.95	.366	45	291.55	.386
50	341.60	.329	50	322.59	.349	50	305.60	.368	50	290.33	.388
55	339.93	.331	55	321.10	.350	55	304.27	.370	55	289.13	.390
17° 00	338.27	.333	18° 00	319.62	.352	19° 00	302.94	.371	20° 00	287.94	.392

TABLE FOR LAYING OUT CURVES

CHORDS GIVEN FOR 5.355 FROM CENTER OF TRACK

Center	15′ Chord on Center			25′ Chord on Center			50′ Chord on Center		
Rad.	Deflection	3′ in.	3′ out	Deflection	3′ in.	3′ out	Deflection	3′ in	3′ out
40	10 48 25	12.99	17.01	18 12 36	21.65	28.35	38 40 57	43.30	56.70
42.355	10 11 58	13.10	16.90	17 09 54	21.83	28.17	36 10 30	43.67	56.33
45.	9 35 39	13.22	16.78	16 07 40	22.02	27.98	33 44 56	44.05	55.95
47.355	9 06 46	13.30	16.70	15 18 20	22.27	27.73	31 52 25	44.35	55.65
50.	8 37 37	13.39	16.61	14 28 39	22.32	27.68	30 00 00	44.65	55.35
52.355	8 14 10	13.46	16.54	13 48 47	22.44	27.56	28 27 30	44.88	55.12
55.	7 50 15	13.54	16.46	13 08 11	22.56	27.44	27 02 08	45.13	54.87
57.355	7 30 50	13,60	16.40	12 35 16	22.66	27.34	25 50 30	45.33	54.67
60.	7 10 51	13.66	16.34	12 01 28	22.77	27.23	24 37 28	45.54	54.46
62.355	6 54 29	13.71	16.29	11 33 51	22.85	27.15	23 38 11	45.71	54.29
65.	6 37 33	13.76	16.24	11 05 15	22.94	27.06	22 37 11	45.88	54.12
67.355	6 23 55	13.81	16.19	10 41 43	23.01	26.99	21 47 16	46.02	53.98
70.	6 09 02	13.85	16.15	10 17 11	23.08	26.92	20 55 30	46.17	53.83
72.355	5 56 59	13.89	16.11	9 56 54	23.15	26.85	20 12 48	46.30	53.70
75.	5 44 21	13.93	16.07	9 35 37	23.22	26.78	19 28 16	46.43	53.57
77.355	5 33 50	13.96	16.04	9 31 05	23.27	26.73	18 51 20	46.54	53.46
80.	5 22 02	14.00	16.00	8 59 21	23.32	26.68	18 12 36	46.65	53.35
82.355	5 13 30	14.02	15.98	8 43 47	23.37	26.63	17 40 18	46.75	53.25
85.	5 03 31	14.05	15.95	8 27 22	23.42	26.58	17 06 17	46.85	53.15
87.355	4 55 31	14.08	15.92	8 12 28	23.47	26.53	16 37 48	46.94	53.06
90.	4 46 48	14.11	15.89	7 59 01	23.51	26.49	16 07 30	47.02	52.98
92.355	4 39 29	14.13	15.87	7 46 43	23.55	26.45	15 42 40	47.10	52.90
95.	4 31 41	14.15	15.85	7 33 39	23.59	26.41	15 15 27	47.18	52.82
97.355	4 25 06	14.17	15.83	7 22 37	23.63	26.37	14 52 47	47.25	52.75
100.	4 18 04	14.20	15.80	7 10 52	23.66	26.34	14 28 39	47.32	52.68
102.355	4 12 08	14.22	15.78	7 00 53	23.69	26.31	14 08 14	47.38	52.62

TABLE FOR LAYING OUT CURVES (*continued*)

CHORDS GIVEN FOR 5.355 FROM CENTER OF TRACK

Center	25′ Chord on Center			50′ Chord on Center			75′ Chord on Center		
Rad.	Deflection	3′ in.	3′ out	Deflection	3′ in.	3′ out	Deflection	3′ in.	3′ out
125	5 44 21	23.93	26.07	11 32 13	47.86	52.14	17 27 23	71.79	78.21
150	4 46 47	24.11	25.89	9 35 37	48.21	51.79	14 28 39	72.32	77.68
175	4 05 46	24.23	25.77	8 12 48	48.47	51.53	12 22 25	72.71	77.29
200	3 35 00	24.33	25.67	7 10 52	48.66	51.34	10 48 25	73.00	77.00
225	3 11 05	24.40	25.60	6 22 47	48.81	51.19	9 35 39	73.22	76.78
250	2 52 02	24.46	25.54	5 44 21	48.93	51.07	8 37 37	73.40	76.60
275	2 36 19	24.51	25.49	5 12 57	49.03	50.97	7 50 15	73.54	76.46
300	2 23 14	24.55	25.45	4 46 47	49.11	50.89	7 10 51	73.66	76.34
325	2 12 15	24.59	25.41	4 24 42	49.18	50.82	6 37 33	73.77	76.23
350	2 02 48	24.62	25.38	4 05 45	49.24	50.76	6 09 02	73.85	76.15
375	1 54 37	24.64	25.36	3 49 21	49.29	50.71	5 44 21	73.93	76.07
400	1 47 27	24.67	25.33	3 35 00	49.33	50.67	5 22 46	74.00	76.00
450	1 35 30	24.70	25.30	3 11 05	49.40	50.60	4 46 49	74.11	75.89
500	1 25 57	24.73	25.27	2 52 02	49.46	50.54	4 18 04	74.20	75.80
550	1 18 08	24.76	25.24	2 36 19	49.51	50.49	3 54 34	74.27	75.73
600	1 11 37	24.78	25.22	2 23 14	49.55	50.45	3 35 00	74.33	75.67
650	1 06 07	24.79	25.21	2 12 15	49.59	50.41	3 18 27	74.38	75.62
700	1 01 24	24.81	25.19	2 02 48	49.62	50.38	3 04 15	74.43	75.57
750	0 57 18	24.82	25.18	1 54 37	49.64	50.36	2 51 58	74.46	75.54
500	0 53 43	24.83	25.17	1 47 27	49.67	50.33	2 41 12	74.50	75.50
850	0 50 33	24.84	25.16	1 41 07	49.69	50.31	2 31 43	74.53	75.47
900	0 47 45	24.85	25.15	1 35 30	49.70	50.30	2 23 37	74.55	75.45
950	0 45 14	24.86	25.14	1 30 28	49.71	50.28	2 15 44	74.58	75.42
1000	0 42 59	24.87	24.13	1 25 58	49.73	50.27	2 08 57	74.60	75.40
1100	0 39 04	24.88	25.12	1 18 08	49.76	50.24	1 57 13	74.63	75.37
1200	0 35 48	24.89	25.11	1 11 37	49.78	50.22	1 47 27	74.67	75.33
1300	0 33 03	24.90	25.10	1 06 07	49.79	50.21	1 39 11	74.69	75.31
1400	0 30 42	24.90	25.10	1 01 24	49.81	50.19	1 32 06	74.71	75.29
1500	0 28 39	24.91	25.09	0 57 18	49.82	50.18	1 25 57	74.73	75.27
1600	0 26 51	24.92	25.08	0 53 43	49.83	50.17	1 20 35	74.75	75.25
1700	0 25 15	24.92	25.08	0 50 30	49.84	50.16	1 15 50	74.76	75.24
1800	0 23 31	24.93	25.07	0 47 43	49.85	50.15	1 11 38	74.78	75.22
2000	0 21 29	24.93	25.07	0 42 59	49.87	50.13	1 04 27	74.80	75.20

DEFLECTION FOR 3-FT. CHORDS

	0°	1°	2°		0°	1°	2°
0		85.94	42.97	30	171.89	57.30	34.38
1	5156.7	84.54	42.62	31	166.35	56.67	34.15
2	2578.3	83.17	42.27	32	161.15	56.05	33.93
3	1718.9	81.85	41.93	33	156.26	55.45	33.71
4	1289.2	80.57	41.59	34	151.67	54.86	33.49
5	1031.3	79.33	41.25	35	147.33	54.28	33.27
6	859.5	78.13	40.92	36	143.24	53.72	33.05
7	736.66	76.96	40.60	37	139.37	53.16	32.84
8	644.58	75.83	40.27	38	135.70	52.62	32.64
9	572.97	74.74	39.98	39	132.22	52.09	32.43
10	515.67	73.67	39.67	40	128.92	51.57	32.23
11	468.80	72.63	39.37	41	125.77	51.06	32.03
12	429.73	71.62	39.07	42	122.78	50.55	31.83
13	396.67	70.64	38.77	43	119.92	50.06	31.64
14	368.33	69.69	38.48	44	117.20	49.58	31.44
15	343.79	68.76	38.20	45	114.59	49.11	31.25
16	322.30	67.85	37.92	46	112.10	48.65	31.06
17	303.33	66.97	37.64	47	109.72	48.20	30.88
18	286.49	66.11	37.37	48	107.43	47.75	30.70
19	271.41	65.28	37.10	49	105.24	47.31	30.51
20	257.83	64.46	36.83	50	103.13	46.88	30.33
21	245.55	63.66	36.57	51	101.11	46.46	30.16
22	234.40	62.89	36.32	52	99.17	46.04	29.98
23	224.20	62.13	36.06	53	97.30	45.63	29.81
24	214.86	61.39	35.81	54	95.49	45.23	29.64
25	206.27	60.67	35.56	55	93.76	44.84	29.47
26	198.34	59.96	35.32	56	92.08	44.45	29.30
27	190.99	59.27	35.08	57	90.47	44.07	29.13
28	184.17	58.60	34.84	58	88.90	43.70	28.97
29	177.82	57.94	34.61	59	87.40	43.33	28.81

FROG SPREADS FOR THREE-FOOT ARMS

1/16″	0′ 3″	0′ 7″	0′ 11″	1′ 3″	1′ 7″	1′ 11″	2′ 3″	2′ 7″	2′ 11″	3′ 3″	3′ 7″	3′ 11″
0	4° 47′	11° 10′	17° 34′	24° 03′	30° 36′	37° 16′	44° 03′	51° 01′	58° 10′	65° 36′	73′ 21°	81° 30′
1	53	16	40	09	42	22	10	07	17	43	28	38
2	59	22	46	15	48	28	16	14	24	50	36	46
3	5 04	28	52	21	55	34	22	20	31	57	43	54
4	10	34	58	27	31 01	41	29	27	38	66 04	50	82 02
5	16	40	18 05	33	07	47	35	34	44	11	58	10
6	22	46	11	40	13	53	42	40	51	18	74 05	18
7	28	52	17	46	19	38 00	48	47	58	26	13	26
8	34	58	23	52	26	06	55	53	59 05	33	20	33
9	40	12 04	29	58	32	12	45 01	52 00	12	40	28	41
10	46	10	35	25 04	38	19	07	07	19	47	35	49
11	52	16	41	10	44	25	14	13	26	54	43	57
12	58	22	47	16	50	31	20	20	33	67 01	50	83.05
13	6 04	28	53	23	57	38	27	27	39	08	58	13
14	10	34	19 00	29	32 03	44	33	33	46	16	75 06	21
15	15	40	06	35	09	50	40	40	53	23	13	29
	0′ 4″	0′ 8″	1′ 0″	1′ 4″	1′ 8″	2′ 0″	2′ 4″	2′ 8″	3′ 0″	3′ 4	3′ 8″	4′ 0″
0	6 22	12 46	19 12	25 41	32 15	38 57	45 46	52 47	60 00	67 30	75 20	83 37
1	28	52	18	47	22	39 03	53	55	07	37	28	45
2	34	58	24	53	28	09	59	53 00	14	44	35	53
3	40	13 04	30	59	34	16	46 06	07	21	52	43	84 01
4	46	10	36	26 05	40	22	12	13	28	59	51	10
5	52	16	42	11	47	28	19	20	35	68 06	58	18
6	58	22	48	17	53	35	25	27	41	13	76 06	26
7	7 04	28	54	24	59	41	32	33	48	20	13	34
8	10	34	20 00	30	33 05	47	38	40	55	27	21	42
9	16	40	06	36	11	54	45	46	61 02	35	29	50
10	22	46	12	42	18	40 00	51	53	09	42	36	58
11	28	52	18	48	24	06	58	54 00	16	49	44	85 06
12	34	58	24	54	30	13	47 04	06	23	56	51	14
13	40	14 04	30	27 00	36	19	11	14	30	69 04	59	22
14	46	10	36	06	43	25	17	20	37	11	77 07	30
15	52	16	42	13	49	32	24	26	44	18	14	38

FROG SPREADS FOR THREE-FOOT ARMS (*continued*)

1/16″	0′ 5″	0′ 9″	1′ 1″	1′ 5″	1′ 9″	2′ 1″	2′ 5″	2′ 9″	3′ 1″	3′ 5″	3′ 9″	4′ 1″
0	7 58	14 22	20 48	27 19	33 55	40 38	47 30	54 33	61 51	69 25	77 22	85 47
1	8 04	28	53	25	34 01	45	37	39	58	33	30	55
2	10	34	21 01	31	07	51	45	46	62 05	40	38	86 03
3	16	40	07	37	14	57	50	53	12	47	45	11
4	22	46	13	43	20	41 04	56	59	19	54	53	19
5	28	52	19	49	26	10	48 03	55 06	26	70 02	78 00	27
6	34	58	25	56	32	16	10	14	33	09	08	36
7	40	15 04	31	28 02	38	23	16	21	40	16	16	44
8	46	10	37	08	45	29	23	27	47	24	23	52
9	52	16	43	14	51	36	29	34	54	31	31	87 00
10	58	22	49	20	57	42	36	41	63 01	38	39	08
11	9 04	28	55	26	35 04	48	42	48	08	46	46	17
12	10	34	22 01	33	10	55	49	54	15	53	54	25
13	16	40	7	39	16	42 01	55	56 02	22	71 00	79 02	23
14	22	46	13	45	22	07	49 02	08	28	08	10	42
15	28	52	19	51	29	14	08	15	36	15	17	50
	0′ 6″	0′ 10″	1′ 2″	1′ 6″	1′ 10″	2′ 2″	2′ 6″	2′ 10″	3′ 2″	3′ 6″	3′ 10″	4′ 2″
0	9 34	15 58	22 25	28 57	35 35	42 20	49 15	56 21	63 43	71 22	79 25	87 58
1	40	16 04	32	29 03	41	27	22	28	50	30	33	06
2	46	10	38	10	48	33	28	35	57	37	41	14
3	52	16	44	16	54	39	35	42	64 04	44	48	23
4	58	22	50	22	36 00	46	41	49	11	52	56	31
5	10 04	28	56	28	06	52	48	56	18	59	80 04	40
6	10	34	23 02	34	13	59	55	57 02	25	72 06	12	48
7	16	40	08	40	19	43 05	50 01	09	32	14	20	56
8	22	46	14	47	25	12	08	17	39	22	28	89 05
9	28	52	20	53	32	18	14	23	46	29	35	13
10	34	58	26	59	38	25	21	29	53	36	43	21
11	40	17 04	32	30 05	44	31	28	36	65 00	43	51	30
12	46	10	39	11	50	37	34	43	07	51	59	38
13	52	16	45	17	57	44	41	50	13	59	81 07	47
14	58	22	51	24	37 03	50	47	57	22	73 06	14	55
15	11 04	28	57	30	09	57	54	58 03	29	13	22	90 04

RIGHT-ANGLED TRIANGLES. ALTITUDE = 4′ 8½″, BASE GIVEN IN FEET AND DECIMALS FOR EACH VARIATION OF $\frac{1}{16}$″, IN HYPOTHENUSE

16ths	4′ 8″	5′ 0″	5′ 4″	5′ 8″	6′ 0″	6′ 4″	6′ 8	7′ 0″	7′ 4″	7′ 8″	8′ 0″	8 4″	8′ 8″	9′ 0″	9′ 4″	9′ 8″
0		1.683	2.505	3.153	3.719	4.236	4.720	5.180	5.622	6.050	6.468	6.875	7.277	7.670	8.059	8.443
1		1.698	2.516	3.163	3.728	4.243	4.727	5.187	5.629	6.057	6.474	6.882	7.283	7.676	8.065	8.449
2		1.714	2.527	3.172	3.736	4.251	4.735	5.194	5.636	6.063	6.481	6.888	7.289	7.682	8.071	8.455
3		1.729	2.538	3.182	3.745	4.259	4.742	5.201	5.643	6.070	6.487	6.894	7.295	7.688	8.077	8.461
4		1.744	2.549	3.191	3.753	4.267	4.749	5.208	5.650	6.076	6.494	6.901	7.301	7.694	8.083	8.466
5		1.758	2.560	3.200	3.762	4.274	4.756	5.215	5.656	6.083	6.500	6.907	7.307	7.701	8.089	8.472
6		1.773	2.571	3.209	3.770	4.282	4.764	5.222	5.663	6.090	6.506	6.913	7.313	7.707	8.095	8.478
7		1.788	2.582	3.218	3.778	4.290	4.771	5.229	5.670	6.096	6.513	6.920	7.319	7.713	8.101	8.484
8		1.803	2.593	3.228	3.786	4.298	4.778	5.236	5.677	6.103	6.519	6.926	7.326	7.719	8.107	8.490
9	0.222	1.817	2.603	3.237	3.795	4.305	4.786	5.243	5.683	6.109	6.526	6.932	7.331	7.725	8.113	8.496
10	0.313	1.832	2.614	3.246	3.803	4.313	4.793	5.250	5.690	6.116	6.532	6.938	7.337	7.731	8.119	8.502
11	0.348	1.846	2.625	3.255	3.811	4.321	4.801	5.257	5.697	6.123	6.539	6.945	7.343	7.737	8.125	8.508
12	0.444	1.860	2.635	3.264	3.819	4.329	4.808	5.264	5.704	6.129	6.545	6.951	7.350	7.743	8.131	8.514
13	0.496	1.874	2.646	3.274	3.828	4.336	4.815	5.271	5.710	6.136	6.551	6.957	7.356	7.749	8.137	8.520
14	0.543	1.889	2.657	3.283	3.836	4.344	4.823	5.278	5.717	6.142	6.558	6.964	7.363	7.756	8.143	8.526
15	0.587	1.903	2.667	3.292	3.844	4.351	4.830	5.285	5.723	6.149	6.564	6.970	7.369	7.762	8.140	8.532
	4′ 9″	5′ 1″	5′ 5″	5′ 9″	6′ 1″	6′ 5″	6′ 9″	7′ 1″	7′ 5″	7′ 9″	8′ 1″	8′ 5″	8′ 9″	9′ 1″	9′ 5″	9′ 9″
0	0.628	1.916	2.678	3.301	3.852	4.359	4.837	5.292	5.730	6.156	6.570	6.976	7.375	7.768	8.155	8.538
1	0.667	1.930	2.688	3.310	3.861	4.367	4.844	5.299	5.737	6.162	6.577	6.983	7.381	7.774	8.161	8.544
2	0.702	1.944	2.699	3.319	3.869	4.374	4.852	5.306	5.743	6.169	6.583	6.989	7.387	7.780	8.167	8.550
3	0.737	1.957	2.709	3.328	3.877	4.382	4.859	5.313	5.750	6.175	6.590	6.995	7.394	7.786	8.173	8.556
4	0.770	1.971	2.720	3.337	3.885	4.390	4.866	5.320	5.757	6.182	6.596	7.001	7.400	7.792	8.179	8.562
5	0.801	1.984	2.730	3.346	3.894	4.398	4.873	5.327	5.764	6.188	6.603	7.008	7.406	7.798	8.185	8.568
6	0.832	1.998	2.741	3.355	3.902	4.405	4.881	5.334	5.771	6.195	6.609	7.014	7.412	7.804	8.191	8.574
7	0.861	2.011	2.751	3.364	3.910	4.413	4.888	5.341	5.777	6.201	6.615	7.020	7.418	7.810	8.197	8.580
8	0.890	2.024	2.761	3.373	3.918	4.421	4.895	5.348	5.784	6.208	6.621	7.027	7.424	7.816	8.203	8.586
9	0.917	2.037	2.772	3.382	3.926	4.428	4.902	5.355	5.791	6.214	6.628	7.033	7.430	7.823	8.209	8.591
10	0.944	2.050	2.782	3.390	3.934	4.436	4.909	5.362	5.797	6.221	6.634	7.039	7.437	7.829	8.215	8.597
11	0.970	2.063	2.792	3.399	3.942	4.443	4.916	5.369	5.803	6.227	6.640	7.046	7.443	7.835	8.221	8.603
12	0.996	2.076	2.802	3.408	3.950	4.451	4.923	5.376	5.810	6.234	6.647	7.052	7.449	7.841	8.227	8.609
13	1.021	2.089	2.813	3.417	3.958	4.458	4.931	5.383	5.817	6.240	6.653	7.058	7.455	7.847	8.233	8.615
14	1.045	2.102	2.823	3.426	3.966	4.466	4.938	5.389	5.824	6.247	6.659	7.064	7.462	7.853	8.239	8.621
15	1.069	2.115	2.833	3.425	3.974	4.473	4.945	5.396	5.831	6.253	6.666	7.070	7.468	7.859	8.245	8.627

RIGHT-ANGLED TRIANGLES. ALTITUDE = 496′ 8½″, BASE GIVEN IN FEET AND DECIMALS FOR EACH VARIATION OF $\frac{1}{16}$″ IN HYPOTHENUSE (*continued*)

16ths	4′ 10″	5′ 2″	5′ 6″	5′ 10″	6′ 2″	6′ 6″	6′ 10″	7′ 2″	7′ 6″	7′ 10″	8′ 2″	8′ 6″	8′ 10″	9′ 2″	9′ 6″	9, 10″
0	1.093	2.127	2.843	3.444	3.982	4.481	4.952	5.403	5.838	6.260	6.673	7.077	7.474	7.865	8.251	8.633
1	1.115	2.140	2.853	3.453	3.991	4.488	4.960	5.410	5.844	6.266	6.679	7.083	7.480	7.871	8.257	8.639
2	1.137	2.153	2.863	3.461	3.999	4.496	4.967	5.417	5.851	6.273	6.685	7.089	7.486	7.878	8.263	8.645
3	1.159	2.165	2.873	3.470	4.007	4.503	4.974	5.424	5.858	6.279	6.692	7.095	7.492	7.884	8.269	8.651
4	1.181	2.177	2.883	3.479	4.015	4.511	4.981	5.431	5.864	6.286	6.698	7.102	7.498	7.890	8.275	8.656
5	1.202	2.190	2.893	3.488	4.023	4.518	4.989	5.437	5.871	6.292	6.704	7.108	7.505	7.896	8.281	8.662
6	1.223	2.202	2.903	3.496	4.031	4.526	4.996	5.444	5.878	6.299	6.711	7.114	7.511	7.902	8.287	8.668
7	1.244	2.215	2.913	3.505	4.039	4.533	5.003	5.451	5.885	6.305	6.717	7.120	7.517	7.908	8.293	8.674
8	1.264	2.227	2.923	3.514	4.047	4.541	5.010	5.458	5.891	6.312	6.723	7.126	7.523	7.914	8.299	8.680
9	1.284	2.239	2.933	3.523	4.055	4.548	5.018	5.465	5.898	6.318	6.730	7.133	7.530	7.920	8.305	8.686
10	1.304	2.251	2.943	3.532	4.063	4.556	5.025	5.472	5.905	6.325	6.736	7.139	7.536	7.926	8.311	8.692
11	1.323	2.263	2.953	3.540	4.071	4.563	5.032	5.479	5.911	6.331	6.743	7.145	7.542	7.932	8.317	8.698
12	1.342	2.275	2.962	3.549	4.079	4.571	5.039	5.486	5.918	6.338	6.749	7.152	7.548	7.938	8.323	8.704
13	1.361	2.287	2.972	3.558	4.087	4.578	5.046	5.493	5.925	6.344	6.756	7.158	7.554	7.944	8.329	8.710
14	1.379	2.299	2.982	3.566	4.094	4.586	5.053	5.500	5.931	6.351	6.762	7.164	7.560	7.950	8.335	8.715
15	1.398	2.311	2.992	3.575	4.102	4.593	5.060	5.506	5.938	6.357	6.768	7.171	7.566	7.956	8.341	8.721

16ths	4′ 11″	5′ 3″	5′ 7″	5′ 11″	6′ 3″	6′ 7	6′ 11″	7′ 3″	7′ 7″	7′11″	8′ 3″	8′ 7″	8′ 11″	9′ 3″	9′ 7″	9′ 11″
0	1.416	2.323	3.001	3.583	4.110	4.601	5.067	5.513	5.945	6.364	6.775	7.177	7.573	7.962	8.347	8.727
1	1.434	2.335	3.011	3.592	4.118	4.608	5.074	5.520	5.951	6.370	6.781	7.183	7.579	7.968	8.353	8.733
2	1.452	2.346	3.020	3.600	4.126	4.616	5.081	5.527	5.958	6.377	6.787	7.189	7.585	7.974	8.359	8.739
3	1.469	2.358	3.030	3.609	4.134	4.623	5.088	5.534	5.965	6.383	6.793	7.195	7.591	7.980	8.365	8.745
4	1.487	2.369	3.039	3.617	4.142	4.631	5.095	5.541	5.971	6.390	6.800	7.202	7.597	7.987	8.371	8.751
5	1.504	2.381	3.049	3.626	4.149	4.638	5.102	5.547	5.978	6.396	6.806	7.208	7.603	7.993	8.377	8.757
6	1.521	2.392	3.058	3.634	4.157	4.646	5.109	5.554	5.985	6.403	6.812	7.214	7.609	7.999	8.383	8.763
7	1.538	2.404	3.068	3.643	4.165	4.653	5.116	5.561	5.991	6.409	6.819	7.220	7.615	8.005	8.389	8.769
8	1.555	2.415	3.078	3.651	4.173	4.661	5.123	5.568	5.998	6.416	6.825	7.227	7.621	8.011	8.395	8.775
9	1.571	2.427	3.087	3.660	4.181	4.668	5.131	5.574	6.004	6.422	6.831	7.233	7.627	8.017	8.401	8.780
10	1.587	2.438	3.097	3.668	4.189	4.676	5.138	5.581	6.011	6.429	6.838	7.239	7.633	8.023	8.407	8.786
11	1.604	2.449	3.106	3.677	4.197	4.683	5.145	5.588	6.017	6.435	6.844	7.245	7.640	8.029	8.413	8.792
12	1.620	2.460	3.116	3.685	4.205	4.691	5.152	5.595	6.024	6.442	6.850	7.251	7.646	8.035	8.419	8.798
13	1.636	2.472	3.125	3.694	4.212	4.698	5.159	5.602	6.030	6.448	6.857	7.257	7.652	8.041	8.425	8.804
14	1.652	2.483	3.135	3.702	4.220	4.706	5.166	5.609	6.037	6.455	6.863	7.264	7.658	8.047	8.431	8.810
15	1.667	2.494	3.144	3.711	4.228	4.713	5.173	5.615	6.043	6.461	6.869	7.270	7.664	8.053	8.437	8.816

RIGHT-ANGLED TRIANGLES. ALTITUDE = 4′ 8½″, BASE GIVEN IN FEET AND DECIMALS FOR EACH VARIATION OF ¼″ IN HYPOTHENUSE

¼	10′ 0″	10′ 7″	11′ 2″	11′ 9″	12′ 4″	12′ 11″	13′ 6″	14′ 1″	14′ 8″	15′ 3″	15′ 10″	16′ 5″	17′ 0″	17′ 7″	18′ 2″	18′ 9″
0	8.822	9.478	10.125	10.765	11.399	12.028	12.652	13.273	13.890	14.505	15.117	15.727	16.335	16.941	17.546	18.149
1	8.845	9.502	10.148	10.787	11.421	12.050	12.674	13.295	13.912	14.527	15.138	15.748	16.356	16.962	17.567	18.171
2	8.869	9.525	10.171	10.810	11.444	12.073	12.697	13.317	13.934	14.549	15.160	15.770	16.378	16.984	17.589	18.102
3	8.892	9.548	10.194	10.833	11.466	12.095	12.719	13.339	13.956	14.571	15.182	15.792	16.400	17.006	17.510	18.214
	10′ 1″	10′ 8″	11′ 3″	11′ 10″	12′ 5″	13′ 0″	13′ 7″	14′ 2″	14′ 9″	15′ 4″	15′ 11″	16′ 6″	17′ 1″	17′ 8″	18′ 3″	18′ 10″
0	8.916	9.571	10.217	10.856	11.489	12.117	12.741	13.361	13.978	14.593	15.204	15.814	16.422	17.028	17.632	18.235
1	8.940	9.595	10.240	10.878	11.511	12.140	12.763	13.383	14.000	14.615	15.225	15.835	16.443	17.049	17.654	18.257
2	8.964	9.618	10.263	10.901	11.534	12.162	12.786	13.406	14.022	14.636	15.247	15.857	16.465	17.071	17.675	18.278
3	8.987	9.641	10.286	10.924	11.556	12.185	12.808	13.428	14.044	14.658	15.269	15.879	16.486	17.092	17.697	18.300
	10′ 2″	10′ 9″	11′ 4″	11′ 11″	12′ 6″	13′ 1″	13′ 8″	14′ 3″	14′ 10″	15′ 5″	16′ 0″	16′ 7″	17 2″	17′ 9″	18′ 4″	18′ 11″
0	9.011	9.664	10.309	10.947	11.579	12.207	12.830	13.450	14.066	14.680	15.291	15.901	16.508	17.114	17.719	18.321
1	9.034	9.688	10.332	10.970	11.601	12.229	12.852	13.472	14.088	14.701	15.313	15.922	16.530	17.135	17.740	18.343
2	9.058	9.711	10.365	10.992	11.624	12.252	12.874	13.494	14.110	14.723	15.335	15.944	16.551	17.157	17.762	18.365
3	9.081	9.734	10.388	11.015	11.646	12.274	12.896	13.516	14.132	14.745	15.357	15.966	16.573	17.178	17.783	18.386
	10′ 3″	10′ 10″	11′ 5″	12′ 0″	12′ 7″	13′ 2″	13′ 9″	14′ 4″	14′ 11″	15′ 6″	16′ 1″	16′ 8″	17′ 3″	17′ 10″	18′ 5″	19′ 0″
0	9.105	9.757	10.401	11.038	11.669	12.296	12.918	13.538	14.154	14.767	15.379	15.988	16.595	17.200	17.805	18.407
1	9.128	9.780	10.423	11.060	11.691	12.318	12.941	13.560	14.176	14.789	15.400	16.009	16.616	17.221	17.827	18.429
2	9.152	9.803	10.446	11.083	11.714	12.341	12.963	13.582	14.198	14.811	15.422	16.031	16.638	17.243	17.848	18.450
3	9.175	9.826	10.469	11.106	11.736	12.363	12.985	13.604	14.220	14.833	15.444	16.052	16.660	17.264	17.870	18.472
	10′ 4″	10′ 11″	11′ 6″	12′ 1″	12′ 8″	13′ 3″	13′ 10″	14′ 5″	15′ 0″	15′ 7″	16′ 2″	16′ 9″	17′ 4″	17′ 11″	18′ 6″	19′ 1″
0	9.198	9.849	10.492	11.129	11.759	12.385	13.007	13.626	14.242	14.855	15.466	16.074	16.682	17.286	17.892	18.493
1	9.222	9.873	10.514	11.151	11.781	12.408	13.030	13.648	14.264	14.876	15.487	16.096	16.703	17.308	17.913	18.515
2	9.245	9.896	10.537	11.174	11.804	12.430	13.052	13.670	14.285	14.898	15.509	16.117	16.725	17.329	17.935	18.536
3	9.268	9.919	10.560	11.196	11.836	12.452	13.074	13.692	14.307	14.920	15.531	16.139	16.746	17.351	17.956	18.558
	10′ 5″	11′ 0″	11′ 7″	12′ 2″	12′ 9″	13′ 4″	13′ 11″	14′ 6″	15′ 1″	15′ 8″	16′ 3″	16′ 10″	17′ 5″	18′ 0″	18′ 7″	19′ 2″
0	9.292	9.942	10.583	11.219	11.849	12.474	13.096	13.714	14.329	14.942	15.553	16.161	16.768	17.373	17.978	18.579
1	9.315	9.965	10.605	11.241	11.871	12.497	13.119	13.736	14.351	14.964	15.574	16.182	16.789	17.394	17.999	18.601
2	9.339	9.988	10.628	11.264	11.894	13.519	13.141	13.758	14.373	14.986	15.596	16.204	16.811	17.416	18.020	18.623
3	9.362	10.011	10.651	11.286	11.916	12.541	13.163	13.780	14.395	15.008	15.618	16.226	16.832	17.437	18.042	18.644
	10′ 6″	11′ 1″	11′ 8″	12′ 3″	12′ 10″	13′ 5″	14′ 0″	14′ 7″	15′ 2″	15′ 9″	16′ 4″	16′ 11″	17′ 6″	18′ 1″	18′ 8″	19′ 3″
0	9.385	10.034	10.674	11.309	11.939	12.563	13.185	13.802	14.417	15.030	15.640	16.248	16.854	17.459	18.063	18.665
1	9.409	10.057	10.696	11.331	11.961	12.585	13.207	13.824	14.439	15.051	15.661	16.269	16.875	17.481	18.035	18.686
2	9.432	10.080	10.719	11.354	11.984	12.608	13.229	13.846	14.461	15.073	15.683	16.291	16.897	17.502	18.106	18.707
3	9.455	10.102	10.742	11.376	12.006	12.630	13.251	13.868	14.483	15.095	15.705	16.313	16.819	17.524	18.128	18.729

TABLE OF TRANSITION CURVES

No.	log. $\frac{a}{r}$	log. $\frac{b}{r}$	θ	log. $\frac{x}{R}$	log. $\frac{y}{R}$	log. $\frac{L.T.}{R}$	log. $\frac{S.T.}{R}$	Def.	log. $\frac{L.C.}{R}$	log. vers θ	Log. sin θ
	10′ Transition			log. arc = 7.4637262				First radius not less than 1000 ft.			
1			0° 10′	4.6264220	7.4637255			0° 05′	7.4637260		
2			0 30	5.3253104	7.7647520			0 12	7.7647549		
3			1 00	5.7725417	7.9408316			0 23	7.9408416		
4			1 40	6.1035301	8.0657444			0 38	8.0657704		
5	6.647617	8.6397735	2 30	6.3667404	8.1626044	7.9643691	7.7270608	0 55	8.1626600	6.9785358	8.6396796
6	6.9184130	8.7867624	3 30	6.5853592	8.2417009	8.0473909	7.7996839	1 16	8.2418006	7.2708258	8.7856753
7	7.1526552	8.9106346	4 40	6.7723510	8.3085142	8.1171008	7.8619471	1 40	8.3086980	7.5204982	8.9104039
8	7.3586143	9.0208069	6 00	6.9357138	8.3663085	8.1772072	7.9164792	2 07	8.3666073	7 7386304	9.0192346
9	7.5425640	9.1166040	7 30	7.0807347	8.4171812	8.2300640	7.9650370	2 38	8.4176417	7.9322270	9.1156997
	20′ Transition			log arc = 7.7647562				First radius not less than 500 ft.			
1			0 20	5.2284810	7.7647537			0 10	7.7647555		
2			1 00	5.9274217	8.0657690			0 25	8.0657805		
3			2 00	6.3745651	8.2418145			0 47	8.2418555		
4	6.9273704	8.7646669	3 20	6.7054959	8.3666490	8.1627874	7.9409848	1 15	8.3667524	7.2283598	8.7645111
5	7.2495017	8.9406656	5 00	6.9686043	8.4633596	8.2655574	8.0283083	1 50	8.4635820	7.5803892	8.9402960
6	7.5206564	9.0866440	7 00	7.1870551	8.5422011	8.3487142	8.1012606	2 32	8.5426238	7.8723806	9.0858945
7	7.7542733	9.2113557	9 20	7.3737798	8.6086143	8.4186369	8.1637881	3 20	8.6093494	8.1218378	9.2099917
8	7.9600277	9.3201733	12 00	7.5367427	8.6658151	8.4790592	8.2188638	4 15	8.6670102	8.3394992	9.3178789
9	8.1436395	9.4166315	15 00	7.6811896	8.7158391	8.5323675	8.2681934	5 17	8.7176825	8.5324254	9.4129962
	30′ Transition			log. arc = 7.9408475				First radius not less than 330 ft.			
1			0 30	5.5806620	7.9408419			0 15	7.9408460		
2			1 30	6.2796213	8.2418386			0 37	8.2418644		
3			3 00	6.7267109	8.4178271			1 10	8.4179171		
4	7.2794730	8.9406212	5 00	7.0575482	8.5425312	8.3388896	8.1172522	1 52	8.5427638	7.5803892	8.9402960
5	7.5308142	9.1175300	7 30	7.3204908	8.6389927	8.4418946	8.2047932	2 45	8.6394930	7.9322270	9.1156977
6	7.8712396	9.2623220	10 30	7.5386554	8.7174097	8.5252833	8.2780224	3 47	8.7183605	8.2238876	9.2606330
7	8.1058316	9.3867497	14 00	7.7249276	8.7831552	8.5855653	8.3412524	5 00	8.7848097	8.4728190	9.3836752
8	8.3111647	9.4951597	18 00	7.8812155	8.8393662	8.6565342	8.3972331	6 22	8.8420562	8.6896948	9.4899824
9	8.4942228	9.5910539	22 30	8.0307022	8.8879961	8.7105974	8.4478625	7 55	8.8921461	8.8815014	9.5828397

TABLE OF THROWS — SWITCHES. DISTANCE ON CHORD AND BASE

Rad.	2′	3′	4′	5′	6′	7′	8′	9′	10′	11′	12′	13	14′	15′	16′	17′
27.645	0 0 14	0 1 15	0 3 7	0 5 7	0 7 14	0 10 12	1 1 15	1 5 10	1 9 12	2 2 6	2 7 7					
32.355	0 0 12	0 1 11	0 2 15	0 4 10	0 6 11	0 9 1	0 11 12	1 3 1	1 6 10	1 10 8	2 2 13					
30.000	0 0 13	0 1 13	0 3 3	0 5 0	0 7 3	0 9 13	1 0 13	1 4 4	1 8 1	2 0 5	2 4 15					
34.710	0 0 11	0 1 9	0 2 12	0 4 5	0 6 4	0 8 8	0 10 12	1 2 0	1 5 6	1 9 0	2 1 0					
32.645	0 0 12	0 1 11	0 2 15	0 4 10	0 6 10	0 9 0	0 11 12	1 2 14	1 6 7	1 10 5	2 2 9					
37.355	0 0 10	0 1 7	0 2 9	0 4 0	0 5 12	0 7 13	0 10 5	1 1 0	1 4 2	1 7 8	1 11 3					
35.	0 0 11	0 1 9	0 2 12	0 4 5	0 6 3	0 8 6	0 11 0	1 1 15	1 5 3	1 8 13	2 0 12					
39.71	0 0 10	0 1 6	0 2 7	0 3 13	0 5 7	0 7 7	0 9 11	1 0 4	1 3 2	1 6 6	1 9 13					
37.645	0 0 10	0 1 7	0 2 9	0 4 0	0 5 12	0 7 13	0 10 3	1 0 15	1 4 0	1 7 8	1 11 1					
42.355	0 0 9	0 1 4	0 2 4	0 3 9	0 5 2	0 6 15	0 9 1	0 11 8	1 2 3	1 5 3	1 8 7					
40.	0 0 10	0 1 6	0 2 6	0 3 12	0 5 6	0 7 5	0 9 10	1 0 2	1 3 0	1 6 3	1 9 11					
44.71	0 0 9	0 1 3	0 2 2	0 3 6	0 4 13	0 6 9	0 8 10	0 10 14	1 1 7	1 4 4	1 7 6					
42.645	0 0 9	0 1 4	0 2 4	0 3 8	0 5 1	0 6 14	0 9 0	0 11 6	1 2 1	1 5 1	1 8 5					
47.355	0 0 8	0 1 2	0 2 1	0 3 3	0 4 9	0 6 4	0 8 2	0 10 5	1 0 11	1 3 5	1 6 5					
45.	0 0 9	0 1 3	0 2 2	0 3 5	0 4 13	0 6 8	0 8 9	0 10 13	1 1 5	1 4 2	1 7 4					
49.71	0 0 8	0 1 1	0 1 15	0 3 0	0 4 6	0 5 15	0 7 13	0 9 13	1 0 1	1 2 10	1 5 7					
47.645	0 0 8	0 1 2	0 2 0	0 3 3	0 4 9	0 6 3	0 8 1	0 10 3	1 0 9	1 3 4	1 6 3	1 9 5	2 0 12	2 4 6		
52.355	0 0 7	0 1 1	0 1 13	0 2 14	0 4 2	0 5 10	0 7 6	0 9 5	0 11 8	1 1 14	1 4 8	1 7 7	1 10 8	2 1 14		
50.	0 0 8	0 1 1	0 1 15	0 3 0	0 4 5	0 5 14	0 7 11	0 9 12	1 0 0	1 2 8	1 5 4	1 8 6	1 11 9	2 3 1		
54.71	0 0 7	0 1 0	0 1 12	0 2 12	0 3 15	0 5 6	0 7 0	0 8 15	0 11 0	1 1 4	1 3 13	1 6 10	1 9 9	2 0 12		
52.645	0 0 8	0 1 1	0 1 13	0 2 14	0 4 2	0 5 10	0 7 5	0 9 4	0 11 6	1 1 13	1 4 7	1 7 5	1 10 6	2 1 12		
57.355	0 0 7	0 0 15	0 1 11	0 2 10	0 3 12	0 5 2	0 6 11	0 8 8	0 10 8	1 0 11	1 3 1	1 5 11	1 8 9	1 11 10		
55.	0 0 7	0 1 0	0 1 12	0 2 12	0 3 15	0 5 6	0 7 0	0 8 14	0 10 15	1 1 3	1 3 11	1 6 7	1 9 7	2 0 10		
59.71	0 0 7	0 0 14	0 1 10	0 2 8	0 3 10	0 4 15	0 6 7	0 8 2	0 10 1	1 0 3	1 2 8	1 5 0	1 7 12	1 10 11		
57.645	0 0 7	0 0 15	0 1 11	0 2 10	0 3 12	0 5 2	0 6 11	0 8 7	0 10 6	1 0 10	1 3 1	1 5 9	1 8 7	1 11 8		
62.355	0 0 6	0 0 14	0 1 9	0 2 6	0 3 8	0 4 12	0 6 3	0 7 13	0 9 10	0 11 10	1 1 14	1 4 4	1 6 14	1 9 12		
60.	0 0 6	0 0 14	0 1 10	0 2 8	0 3 10	0 4 14	0 6 7	0 8 2	0 10 0	1 0 2	1 2 6	1 4 14	1 7 11	1 10 10		
64.71	0 0 6	0 0 13	0 1 8	0 2 5	0 3 5	0 4 9	0 5 15	0 7 8	0 9 5	0 11 4	1 1 6	1 3 11	1 6 3	1 8 15		
62.645	0 0 6	0 0 14	0 1 9	0 2 6	0 3 7	0 4 11	0 6 3	0 7 12	0 9 9	0 11 9	1 1 13	1 4 3	1 6 12	1 9 10		
67.355	0 0 6	0 0 13	0 1 7	0 2 4	0 3 3	0 4 6	0 5 11	0 7 4	0 8 15	0 10 13	1 0 13	1 3 1	1 5 8	1 8 1		
65.	0 0 6	0 0 13	0 1 8	0 2 5	0 3 5	0 4 8	0 5 15	0 7 8	0 9 4	0 11 3	1 1 5	1 3 10	1 6 2	1 8 14		
69.71	0 0 6	0 0 12	0 1 6	0 2 3	0 3 2	0 4 3	0 5 8	0 7 0	0 8 10	0 10 7	1 0 6	1 2 9	1 4 14	1 7 7		

TABLE OF THROWS — SWITCHES. DISTANCE ON CHORD AND BASE (*continued*)

67.645	0 0 6	0 0 13	0 1 7	0 2 4	0 3 3	0 4 5	0 5 11	0 7 3	0 8 14	0 10 12	1 0 12	1 3 0	1 5 6	1 7 15		
72.355	0 0 5	0 0 12	0 1 5	0 2 1	0 3 0	0 4 1	0 5 5	0 6 12	0 8 5	0 10 1	0 11 15	1 2 0	1 4 4	1 6 11		
70.	0 0 5	0 0 12	0 1 6	0 2 3	0 3 1	0 4 3	0 5 8	0 6 15	0 8 9	0 10 6	1 0 6	1 2 8	1 4 13	1 7 5	1 10 0	2 0 12
74.71	0 0 5	0 0 11	0 1 5	0 2 0	0 2 14	0 3 15	0 5 2	0 6 8	0 8 1	0 9 12	0 11 9	1 1 9	1 3 12	1 6 1	1 8 9	1 11 4
72.645	0 0 5	0 0 12	0 1 5	0 2 1	0 3 0	0 4 1	0 5 5	0 6 11	0 8 4	0 10 0	0 11 14	1 1 15	1 4 3	1 6 9	1 9 2	1 11 14
77.355	0 0 5	0 0 11	0 1 4	0 1 15	0 2 13	0 3 13	0 5 0	0 6 5	0 7 12	0 9 6	0 11 3	1 1 2	1 3 3	1 5 7	1 7 14	1 10 7
75.	0 0 5	0 0 11	0 1 5	0 2 0	0 2 14	0 3 15	0 5 2	0 6 8	0 8 0	0 9 11	0 11 8	1 1 8	1 3 11	1 6 0	1 8 8	1 11 2
79.71	0 0 5	0 0 11	0 1 4	0 1 14	0 2 11	0 3 11	0 4 13	0 6 2	0 7 9	0 9 2	0 10 14	1 0 11	1 2 12	1 4 15	1 7 4	1 9 13
77.645	0 0 5	0 0 11	0 1 4	0 1 15	0 2 13	0 3 13	0 4 15	0 6 4	0 7 12	0 9 6	0 11 2	1 1 1	1 3 2	1 5 6	1 7 12	1 10 5
82.355	0 0 5	0 0 11	0 1 3	0 1 13	0 2 10	0 3 9	0 4 11	0 5 15	0 7 6	0 8 13	0 10 8	1 0 5	1 2 4	1 4 7	1 6 11	1 9 1
80.	0 0 5	0 0 11	0 1 3	0 1 14	0 2 11	0 3 11	0 4 13	0 6 1	0 7 8	0 9 1	0 10 13	1 0 11	1 2 11	1 4 14	1 7 3	1 9 11
84.71	0 0 5	0 0 10	0 1 2	0 1 13	0 2 9	0 3 8	0 4 9	0 5 12	0 7 1	0 8 9	0 10 3	1 0 0	1 1 14	1 3 15	1 6 2	1 8 8
82.645	0 0 5	0 0 11	0 1 3	0 1 13	0 2 10	0 3 9	0 4 10	0 5 14	0 7 4	0 8 13	0 10 7	1 0 5	1 2 3	1 4 5	1 6 9	1 9 0
87.355	0 0 5	0 0 10	0 1 2	0 1 12	0 2 8	0 3 6	0 4 6	0 5 9	0 6 14	0 8 5	0 9 14	0 11 10	1 1 7	1 3 7	1 5 9	1 7 14
85.	0 0 5	0 0 10	0 1 2	0 1 12	0 2 9	0 3 7	0 4 8	0 5 12	0 7 1	0 8 9	0 10 3	0 11 15	1 1 13	1 3 14	1 6 1	1 8 6
89.71	0 0 4	0 0 10	0 1 1	0 1 11	0 2 7	0 3 4	0 4 5	0 5 7	0 6 11	0 8 2	0 9 10	0 11 5	1 1 2	1 3 1	1 5 2	1 7 5
87.645	0 0 5	0 0 10	0 1 1	0 1 11	0 2 8	0 3 6	0 4 6	0 5 9	0 6 14	0 8 4	0 9 14	0 11 9	1 1 7	1 3 7	1 5 9	1 7 15
92.355	0 0 4	0 0 10	0 1 1	0 1 10	0 2 6	0 3 3	0 4 3	0 5 4	0 6 8	0 7 14	0 9 6	0 11 0	1 0 12	1 2 10	1 4 10	1 6 13
90.	0 0 4	0 0 10	0 1 1	0 1 11	0 2 7	0 3 4	0 4 4	0 5 6	0 6 11	0 8 1	0 9 10	0 11 4	1 1 1	1 3 0	1 5 1	1 7 5
94.71	0 0 4	0 0 9	0 1 0	0 1 9	0 2 5	0 3 1	0 4 1	0 5 2	0 6 6	0 7 11	0 9 2	0 10 11	1 0 7	1 2 4	1 4 3	1 6 5
92.645	0 0 4	0 0 10	0 1 0	0 1 10	0 2 6	0 3 3	0 4 2	0 5 4	0 6 8	0 7 13	0 9 5	0 10 15	1 0 11	1 2 9	1 4 9	1 6 12
97.355	0 0 4	0 0 9	0 1 0	0 1 9	0 2 4	0 3 0	0 3 15	0 5 0	0 6 3	0 7 7	0 8 14	0 10 7	1 0 1	1 1 14	1 3 13	1 5 13
95.	0 0 4	0 0 9	0 1 0	0 1 9	0 2 5	0 3 2	0 4 1	0 5 2	0 6 5	0 7 10	0 9 1	0 10 11	1 0 6	1 2 3	1 4 3	1 6 4
99.71	0 0 4	0 0 9	0 0 15	0 1 8	0 2 3	0 2 15	0 3 14	0 4 14	0 6 0	0 7 5	0 8 11	0 10 3	0 11 13	1 1 9	1 3 7	1 5 6
97.645	0 0 4	0 0 9	0 0 15	0 1 9	0 2 3	0 3 0	0 3 15	0 5 0	0 6 2	0 7 7	0 8 14	0 10 6	1 0 0	1 1 13	1 3 12	1 5 12
102.355	0 0 4	0 0 9	0 0 15	0 1 7	0 2 2	0 2 14	0 3 12	0 4 12	0 5 14	0 7 1	0 8 7	0 9 14	0 11 8	1 1 3	1 3 0	1 4 15
100.	0 0 4	0 0 9	0 0 15	0 1 8	0 2 3	0 2 15	0 3 13	0 4 14	0 6 0	0 7 4	0 8 10	0 10 2	0 11 12	1 1 8	1 3 6	1 5 6
104.71	0 0 4	0 0 8	0 0 15	0 1 7	0 2 1	0 2 13	0 3 11	0 4 10	0 5 12	0 6 15	0 8 14	0 9 11	0 11 14	0 10 14	1 2 11	1 4 9
147.645	0 0 3	0 0 6	0 0 10	0 1 0	0 1 7	0 2 0	0 2 10	0 3 5	0 4 1	0 4 15	0 5 14	0 6 14	0 7 15	0 9 2	0 10 7	0 11 12
152.355	0 0 3	0 0 6	0 0 10	0 1 0	0 1 7	0 1 15	0 2 8	0 3 3	0 3 15	0 4 12	0 5 11	0 6 11	0 7 12	0 8 14	0 10 1	0 11 6
150.	0 0 3	0 0 6	0 0 10	0 1 0	0 1 7	0 1 15	0 2 9	0 3 4	0 4 0	0 4 14	0 5 12	0 6 12	0 7 13	0 9 0	0 10 4	0 11 9
154.71	0 0 3	0 0 6	0 0 10	0 1 0	0 1 6	0 1 14	0 2 7	0 3 2	0 3 14	0 4 11	0 5 9	0 6 9	0 7 10	0 8 12	0 9 15	0 11 3
197.645	0 0 2	0 0 5	0 0 8	0 0 12	0 1 1	0 1 8	0 1 15	0 2 7	0 3 1	0 3 11	0 4 6	0 5 2	0 5 15	0 6 13	0 7 12	0 8 13
202.355	0 0 2	0 0 5	0 0 8	0 0 12	0 1 1	0 1 7	0 1 14	0 2 6	0 2 15	0 3 9	0 4 4	0 5 0	0 5 13	0 6 11	0 7 9	0 8 9
200.	0 0 2	0 0 5	0 0 8	0 0 12	0 1 1	0 1 8	0 1 15	0 2 7	0 3 0	0 3 10	0 4 5	0 5 1	0 5 14	0 6 12	0 7 11	0 8 11
204.71	0 0 2	0 0 5	0 0 8	0 0 12	0 1 1	0 1 7	0 1 14	0 2 6	0 2 15	0 3 8	0 4 4	0 4 15	0 5 12	0 6 10	0 7 8	0 8 8

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